

Precalculus | North Dakota Mathematics K-12 Standards Correlation to Eureka Math®

About Eureka Math

Created by Great Minds[®], a mission-driven Public Benefit Corporation, *Eureka Math*[®] helps teachers deliver unparalleled math instruction that provides students with a deep understanding and fluency in math. Crafted by teachers and math scholars, the curriculum carefully sequences the mathematical progressions to maximize coherence from Prekindergarten through Precalculus–a principle tested and proven to be essential in students' mastery of math.

Teachers and students using *Eureka Math* find the trademark "Aha!" moments in *Eureka Math* to be a source of joy and inspiration, lesson after lesson, year after year.

Aligned

Great Minds offers detailed analyses that demonstrate how each grade of *Eureka Math* aligns with specific state standards. Access these free alignment studies at <u>greatminds.org/state-studies</u>.

Data

Schools and districts nationwide are experiencing student growth and impressive test scores after using *Eureka Math*. See their stories and data at greatminds.org/data.

Full Suite of Resources

Great Minds offers the *Eureka Math* curriculum as PDF downloads for free, noncommercial use. Access the free PDFs at <u>greatminds.org/</u><u>math/curriculum</u>.

The teacher-writers who created the curriculum have also developed essential resources, available only from Great Minds, including the following:

- Printed material in English and Spanish
- Digital resources
- Professional development
- Classroom tools and manipulatives
- Teacher support materials
- Parent resources

Math Attributes	Aligned Components of Eureka Math
9-12.MA.P Learners can analyze, execute, critique, and adapt approaches and solutions when problem-solving in novel situations.	Lessons in every module engage students in math attributes. These are indicated in margin notes included with every lesson.
9-12.MA.C Learners can create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.	Lessons in every module engage students in math attributes. These are indicated in margin notes included with every lesson.
9-12.MA.R Learners can reason logically, citing evidence to critique and explain what they see, think, and conclude through exploration, generalization, and validation.	Lessons in every module engage students in math attributes. These are indicated in margin notes included with every lesson.

Number and Operations: Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

North Dakota Mathematics K–12 Standards	Aligned Components of Eureka Math
11-12.NO.6	Precalculus and Advanced Topics M1 Lesson 5: An Appearance of Complex Numbers
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. Understand the hierarchal relationships among subsets of the complex number system.	
11-12.NO.7	Precalculus and Advanced Topics M1 Lesson 4: An Appearance of Complex Numbers
Use the definition $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Precalculus and Advanced Topics M1 Lesson 5: An Appearance of Complex Numbers
11-12.NO.8	Precalculus and Advanced Topics M1 Lesson 7: Complex Number Division
Use conjugates to find quotients of complex numbers.	Precalculus and Advanced Topics M1 Lesson 8: Complex Number Division
	Precalculus and Advanced Topics M1 Lesson 9: The Geometric Effect of Some Complex Arithmetic
	Precalculus and Advanced Topics M1 Lesson 17: The Geometric Effect of Multiplying by a Reciprocal
11-12.NO.9	Precalculus and Advanced Topics M3 Lesson 1: Solutions to Polynomial Equations
Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions. Find all solutions to a polynomial equation.	Precalculus and Advanced Topics M3 Lesson 2: Does Every Complex Number Have a Square Root?
	Precalculus and Advanced Topics M3 Lesson 3: Roots of Unity

11-12.NO.10	Precalculus and Advanced Topics M1 Lesson 4: An Appearance of Complex Numbers
Represent complex numbers	Precalculus and Advanced Topics M1 Lesson 5: An Appearance of Complex Numbers
on the complex plane in rectangular, trigonometric, and polar forms. Find the	Precalculus and Advanced Topics M1 Lesson 6: Complex Numbers as Vectors
modulus (absolute value) of a complex	Precalculus and Advanced Topics M1 Lesson 8: Complex Number Division
number. Explain why the rectangular,	Precalculus and Advanced Topics M1 Topic B: Complex Number Operations and Transformations
trigonometric, and polar forms of a given complex number represent the	Precalculus and Advanced Topics M1 Lesson 18: Exploiting the Connection to Trigonometry
same number.	Precalculus and Advanced Topics M1 Lesson 19: Exploiting the Connection to Trigonometry
	Precalculus and Advanced Topics M1 Lesson 20: Exploiting the Connection to Cartesian Coordinates
11-12.NO.11	Precalculus and Advanced Topics M1 Lesson 6: Complex Numbers as Vectors
Represent addition, subtraction,	Precalculus and Advanced Topics M1 Lesson 9: The Geometric Effect of Some Complex Arithmetic
multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.	Precalculus and Advanced Topics M1 Lesson 10: The Geometric Effect of Some Complex Arithmetic
	Precalculus and Advanced Topics M1 Lesson 14: Discovering the Geometric Effect of Complex Multiplication
	Precalculus and Advanced Topics M1 Lesson 15: Justifying the Geometric Effect of Complex Multiplication
	Precalculus and Advanced Topics M1 Lesson 16: Representing Reflections with Transformations
	Precalculus and Advanced Topics M1 Lesson 17: The Geometric Effect of Multiplying by a Reciprocal
	Precalculus and Advanced Topics M1 Lesson 18: Exploiting the Connection to Trigonometry
	Precalculus and Advanced Topics M1 Lesson 19: Exploiting the Connection to Trigonometry
	Precalculus and Advanced Topics M1 Lesson 20: Exploiting the Connection to Cartesian Coordinates
11-12.NO.12	Precalculus and Advanced Topics M3 Lesson 1: Solutions to Polynomial Equations
Extend polynomial identities to the	Precalculus and Advanced Topics M3 Lesson 2: Does Every Complex Number Have a Square Root?
complex numbers.	Precalculus and Advanced Topics M3 Lesson 3: Roots of Unity

North Dakota Mathematics
K–12 Standards

11-12.NO.14 Recognize vector quantities as having both magnitude and direction, writing them in polar form.	Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges
11-12.NO.15 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Precalculus and Advanced Topics M2 Lesson 19: Directed Line Segments and Vectors
11-12.NO.16 Solve problems involving magnitude and direction that can be represented by vectors.	Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges Precalculus and Advanced Topics M2 Lesson 23: Why Are Vectors Useful?

11-12.NO.17

Add and subtract vectors.

- a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Know that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- c. Understand that vector subtraction v w is defined as v + (-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction.

Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction.

Aligned Components of Eureka Math

Precalculus and Advanced Topics M2 Lesson 5: Coordinates of Points in Space Precalculus and Advanced Topics M2 Lesson 6: Linear Transformations as Matrices Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps Precalculus and Advanced Topics M2 Lesson 19: Directed Line Segments and Vectors Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges Precalculus and Advanced Topics M2 Lesson 23: Why Are Vectors Useful? Precalculus and Advanced Topics M2 Lesson 24: Why Are Vectors Useful? Т

North Dakota Mathematics	
K–12 Standards	

Aligned Components of Eureka Math

11-12.NO.18	Precalculus and Advanced Topics M2 Lesson 5: Coordinates of Points in Space
Multiply a vector by a scalar.	Precalculus and Advanced Topics M2 Lesson 6: Linear Transformations as Matrices
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Use the components to perform scalar multiplication (e.g., as $c(v_x, v_y) = (cv_x, cv_y)$).	Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges
b. Compute the magnitude of a scalar multiple cv using $ cv = c v$.	
c. Compute the direction of cv knowing that when $ c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).	
11-12.NO.19	Precalculus and Advanced Topics M1 Lesson 22: Modeling Video Game Motion with Matrices
Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is not commutative.	 Precalculus and Advanced Topics M1 Lesson 24: Matrix Notation Encompasses New Transformations! Precalculus and Advanced Topics M1 Lesson 25: Matrix Multiplication and Addition Precalculus and Advanced Topics M2 Topic A: Networks and Matrices Precalculus and Advanced Topics M2 Topic B: Linear Transformations of Planes and Space Precalculus and Advanced Topics M2 Topic E: First-Person Video Games–Projection Matrices

_

Algebraic Reasoning: Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

North Dakota Mathematics K-12 Standards Aligned Components of Eureka Math Precalculus and Advanced Topics M3 Lesson 10: The Structure of Rational Expressions 11-12.AR.5 Add, subtract, multiply, and divide Precalculus and Advanced Topics M3 Lesson 11: Rational Functions rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. 11-12.AR.11 Supplemental material is necessary to address this standard. Solve quadratic equations with real coefficients that have solutions of the form a + bi and a - bi. 11-12.AR.18 Precalculus and Advanced Topics M2 Topic C: Systems of Linear Equations Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater). Precalculus and Advanced Topics M2 Topic C: Systems of Linear Equations 11-12.AR.19 Solve a system of equations in three or more variables with matrices (using technology).

Aligned Components of Eureka Math

11-12.AR.20	Precalculus and Advanced Topics M3 Lesson 4: The Binomial Theorem
Apply the Binomial Theorem for the expansion of $(ax + by)^n$ in powers of x and y for a positive integer n and integers a and b .	Precalculus and Advanced Topics M3 Lesson 5: The Binomial Theorem

Algebraic Reasoning: Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Functions: Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

North Dakota Mathematics K–12 Standards

11-12.AR.F.1	Precalculus and Advanced Topics M3 Lesson 16: Function Composition
Write a function that describes a relationship between two quantities.	Precalculus and Advanced Topics M3 Lesson 17: Solving Problems by Functions Composition Precalculus and Advanced Topics M4 Lesson 6: Waves, Sinusoids, and Identities
a. Combine standard function types using arithmetic operations.	
b. Compose functions.	

11-12.AR.F.4	Algebra II M1 Lesson 15: Structure in Graphs of Polynomial Functions
Identify the effect of transformations on the graph of a function by replacing f(x) with $af(x)$, $f(bx)$, $f(x - h)$, and f(x) + k, for specific values of a , h , and $k(both positive and negative). Find thevalue of a, b, h, and k given the graphof the function. Recognize even andodd functions from their graphs andequations.$	Algebra II M3 Lesson 20: Transformations of the Graphs of Logarithmic and Exponential Functions Algebra II M3 Lesson 21: The Graph of the Natural Logarithm Function Precalculus and Advanced Topics M3 Lesson 12: End Behavior of Rational Functions Precalculus and Advanced Topics M3 Lesson 15: Transforming Rational Functions Precalculus and Advanced Topics M4 Lesson 11: Revisiting the Graphs of the Trigonometric Functions
11-12.AR.F.5	Precalculus and Advanced Topics M3 Topic C: Inverse Functions
Find inverse functions.	
a. Verify by composition that one function is the inverse of another.	
 Recognize that the graph of a function and its inverse are reflection images over the line y = x. 	
c. Produce an invertible function from a non-invertible function by restricting the domain.	
11-12.AR.F.6	Precalculus and Advanced Topics M3 Lesson 20: Inverses of Logarithmic and Exponential Functions
Apply the inverse relationship between exponents and logarithms to solve problems.	Precalculus and Advanced Topics M3 Lesson 21: Logarithmic and Exponential Problem Solving

11-12.AR.F.11	Algebra II M2 Lesson 8: Graphing the Sine and Cosine Functions
Analyze and graph functions expressed	Algebra II M2 Lesson 11: Transforming the Graph of the Sine Function
symbolically (by hand in simple cases and using technology for more complicated cases), identifying key features	Algebra II M2 Lesson 12: Ferris Wheels–Using Trigonometric Functions to Model Cyclical Behavior
	Algebra II M2 Lesson 14: Graphing the Tangent Function
of the graph.	Precalculus and Advanced Topics M3 Lesson 12: End Behavior of Rational Functions
a. Graph rational functions, identifying domain, range, asymptote(s),	Precalculus and Advanced Topics M3 Lesson 13: Horizontal and Vertical Asymptotes of Graphs of Rational Functions
removable and non-removable discontinuities, intercepts, behavior	Precalculus and Advanced Topics M3 Lesson 14: Graphing Rational Functions
at the asymptote(s), and end	Precalculus and Advanced Topics M3 Lesson 15: Transforming Rational Functions
behavior.	Precalculus and Advanced Topics M4 Lesson 11: Revisiting the Graphs of the Trigonometric Functions
 b. Graph trigonometric functions, showing period, midline, phase shift, and amplitude. 	
11-12.AR.F.15	Precalculus and Advanced Topics M3 Lesson 20: Inverses of Logarithmic and Exponential Functions
Use properties of logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.	Precalculus and Advanced Topics M3 Lesson 21: Logarithmic and Exponential Problem Solving
11-12.AR.F.16	Precalculus and Advanced Topics M4 Lesson 1: Special Triangles and the Unit Circle
Extend right triangle trigonometry and apply knowledge of the unit circle to determine values of sine, cosine, and tangent for multiples of $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.	

11-12.AR.F.19 Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.	Precalculus and Advanced Topics M4 Lesson 1: Special Triangles and the Unit Circle
11-12.AR.F.20 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Precalculus and Advanced Topics M4 Lesson 2: Properties of Trigonometric Functions
11-12. AR.F.21 Create a trigonometric function to model periodic phenomena.	Precalculus and Advanced Topics M4 Lesson 6: Waves, Sinusoids, and Identities
11-12. AR.F.22 Restrict the domain of a trigonometric function to construct its inverse.	Precalculus and Advanced Topics M4 Lesson 12: Inverse Trigonometric Functions
11-12. AR.F.23 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions and interpret them in context.	Precalculus and Advanced Topics M4 Lesson 12: Inverse Trigonometric Functions Precalculus and Advanced Topics M4 Lesson 13: Modeling with Inverse Trigonometric Functions Precalculus and Advanced Topics M4 Lesson 14: Modeling with Inverse Trigonometric Functions
11-12. AR.F.24 Know and apply the addition and subtraction formulas for sine, cosine, and tangent to solve problems.	Precalculus and Advanced Topics M4 Lesson 3: Addition and Subtraction Formulas Precalculus and Advanced Topics M4 Lesson 4: Addition and Subtraction Formulas Precalculus and Advanced Topics M4 Lesson 6: Waves, Sinusoids, and Identities

North Dakota Mathematics

_

Geometry and Measurement: Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

K–12 Standards	Aligned Components of Eureka Math
11-12.GM.1	Geometry M5 Lesson 17: Writing the Equation for a Circle
Write the equation of a conic section given its special features. Convert between the standard form and general form equations of conic sections.	Geometry M5 Lesson 18: Recognizing Equations of Circles
	Algebra II M1 Lesson 33: The Definition of a Parabola
	Algebra II M1 Lesson 34: Are All Parabolas Congruent?
	Algebra II M1 Lesson 35: Are All Parabolas Similar?
	Precalculus and Advanced Topics M3 Lesson 6: Curves in the Complex Plane
	Precalculus and Advanced Topics M3 Lesson 7: Curves from Geometry
	Precalculus and Advanced Topics M3 Lesson 8: Curves from Geometry
11-12.GM.2	Geometry M5 Lesson 17: Writing the Equation for a Circle
Identify key features of a conic section given its equation. Apply properties of conic sections in context.	Geometry M5 Lesson 18: Recognizing Equations of Circles
	Algebra II M1 Lesson 33: The Definition of a Parabola
	Algebra II M1 Lesson 34: Are All Parabolas Congruent?
	Algebra II M1 Lesson 35: Are All Parabolas Similar?
	Precalculus and Advanced Topics M3 Lesson 6: Curves in the Complex Plane
	Precalculus and Advanced Topics M3 Lesson 7: Curves from Geometry
	Precalculus and Advanced Topics M3 Lesson 8: Curves from Geometry
	Supplemental material is necessary to address applying properties of conic sections in context.

K–12 Standards	Aligned Components of Eureka Math
11-12.GM.3	Geometry M2 Lesson 25: Incredibly Useful Ratios
Determine and apply appropriate formulas to solve right and non-right triangle problems in context.	Geometry M2 Lesson 26: The Definition of Sine, Cosine, and Tangent
	Geometry M2 Lesson 28: Solving Problems Using Sine and Cosine
	Geometry M2 Lesson 29: Applying Tangents
	Geometry M2 Lesson 30: Trigonometry and the Pythagorean Theorem
	Geometry M2 Lesson 31: Using Trigonometry to Determine Area
	Geometry M2 Lesson 32: Using Trigonometry to Find Side Lengths of an Acute Triangle
	Geometry M2 Lesson 33: Applying the Laws of Sines and Cosines
	Geometry M2 Lesson 34: Unknown Angles
	Precalculus and Advanced Topics M4 Lesson 8: Law of Sines
	Precalculus and Advanced Topics M4 Lesson 9: Law of Cosines
	Precalculus and Advanced Topics M4 Lesson 10: Putting the Law of Cosines and the Law of Sines to Use
	Supplemental material is necessary to address solving right triangle problems in context.
11-12.GM.4	Geometry M2 Lesson 31: Using Trigonometry to Determine Area
Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Precalculus and Advanced Topics M4 Lesson 7: An Area Formula for Triangles

Data, Probability, and Statistics: Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.

K–12 Standards	Aligned Components of Eureka Math
11-12.DPS.10	Precalculus and Advanced Topics M5 Topic A: Probability
Determine when the order in counting matters and use permutations and combinations to compute probabilities of events accordingly. Determine probability situations as conditional, "or" (union), or "and" (intersection), and determine the probability of an event.	
11-12.DPS.11 Use permutations and combinations to compute probabilities of compound events and solve problems.	Precalculus and Advanced Topics M5 Lesson 4: Using Permutations and Combinations to Compute Probabilities
11-12.DPS.12	Precalculus and Advanced Topics M5 Lesson 5: Discrete Random Variables
Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. Graph the corresponding probability distribution using the same graphical displays as for data distributions.	Precalculus and Advanced Topics M5 Lesson 6: Probability Distribution of a Discrete Random Variable
	Precalculus and Advanced Topics M5 Lesson 13: Games of Chance and Expected Value
	Precalculus and Advanced Topics M5 Lesson 14: Games of Chance and Expected Value
	Precalculus and Advanced Topics M5 Lesson 15: Using Expected Values to Compare Strategies

North Dakota Mathematics

K–12 Standards	Aligned Components of Eureka Math
11-12.DPS.13	Precalculus and Advanced Topics M5 Lesson 7: Expected Value of a Discrete Random Variable
Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	Precalculus and Advanced Topics M5 Lesson 8: Interpreting Expected Value Precalculus and Advanced Topics M5 Lesson 13: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 14: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 15: Using Expected Values to Compare Strategies
 11-12.DPS.14 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values. 	Precalculus and Advanced Topics M5 Lesson 13: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 14: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 15: Using Expected Values to Compare Strategies
11-12.DPS.15 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities are calculated; find the expected value.	Precalculus and Advanced Topics M5 Lesson 9: Determining Discrete Probability Distributions Precalculus and Advanced Topics M5 Lesson 10: Determining Discrete Probability Distributions Precalculus and Advanced Topics M5 Lesson 13: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 14: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 15: Using Expected Values to Compare Strategies

North Dakota Mathematics
K–12 Standards

11-12.DPS.16	Precalculus and Advanced Topics M5 Lesson 11: Estimating Probability Distributions Empirically
Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.	Precalculus and Advanced Topics M5 Lesson 12: Estimating Probability Distributions Empirically Precalculus and Advanced Topics M5 Lesson 13: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 14: Games of Chance and Expected Value Precalculus and Advanced Topics M5 Lesson 15: Using Expected Values to Compare Strategies
11-12.DPS.17 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Precalculus and Advanced Topics M5 Lesson 16: Making Fair Decisions Precalculus and Advanced Topics M5 Lesson 17: Fair Games
11-12.DPS.18 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Precalculus and Advanced Topics M5 Lesson 15: Using Expected Values to Compare Strategies Precalculus and Advanced Topics M5 Lesson 18: Analyzing Decisions and Strategies Using Probability Precalculus and Advanced Topics M5 Lesson 19: Analyzing Decisions and Strategies Using Probability