# EUREKA MATH<sup>™</sup>

ABOUT EUREKA MATH	Created by the nonprofit Great Minds, <i>Eureka Math</i> helps teachers deliver unparalleled math instruction that provides students with a deep understanding and fluency in math. Crafted by teachers and math scholars, the curriculum carefully sequences the mathematical progressions to maximize coherence from Prekindergarten through Precalculus—a principle tested and proven to be essential in students' mastery of math.		
	Teachers and students using <i>Eureka Math</i> find the trademark "Aha!" moments in <i>Eureka Math</i> to be a source of joy and inspiration, lesson after lesson, year after year.		
ALIGNED	<i>Eureka Math</i> is the only curriculum found by EdReports.org to align fully with the Common Core State Standards for Mathematics for all grades, Kindergarten through Grade 8. Great Minds offers detailed analyses which demonstrate how each grade of <i>Eureka Math</i> aligns with specific state standards. Access these free alignment studies at greatminds.org/state-studies.		
DATA	Schools and districts nationwide are experiencing student growth and impressive test scores after using <i>Eureka Math</i> . See their stories and data at greatminds.org/data.		
FULL SUITE OF RESOURCES			
	The teacher–writers who created the curriculum have also developed essential resources, available only from Great Minds, including the following:		
	<ul> <li>Printed material in English and Spanish</li> <li>Digital resources</li> <li>Professional development</li> <li>Classroom tools and manipulatives</li> </ul>		
	Teacher support materials		

• Parent resources

# PRECALCULUS

Many of the Precalculus Tennessee State Mathematics Standards will require the use of *Eureka Math* content from other courses or supplemental materials. A detailed analysis of alignment is provided in the table below. With strategic placement of supplemental materials, *Eureka Math* can ensure students are successful in achieving the proficiencies of the Tennessee State Mathematics Standards while still benefiting from the coherence and rigor of *Eureka Math*.

# **INDICATORS**

Green indicates that the Tennessee standard is fully addressed in *Eureka Math*.

Yellow indicates that the Tennessee standard may not be completely addressed in *Eureka Math*.

Red indicates that the Tennessee standard is not addressed in *Eureka Math*.

Blue indicates there is a discrepancy between the grade level at which this standard is addressed in the Tennessee standards and in *Eureka Math*.

# 1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Lessons in every module engage students in making sense of problems and persevering in solving them as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 1, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M1: Complex Numbers and Transformations

Precalculus and Advanced Topics M4: Trigonometry

### **2:** Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects. Lessons in every module engage students in reasoning abstractly and quantitatively as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 2, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M1: Complex Numbers and Transformations

Precalculus and Advanced Topics M2: Vectors and Matrices

# 3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Lessons in every module engage students in constructing viable arguments and critiquing the reasoning of others as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 3, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M1: Complex Numbers and Transformations

Precalculus and Advanced Topics M3: Rational and Exponential Functions

Precalculus and Advanced Topics M4: Trigonometry

## 4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Lessons in every module engage students in modeling with mathematics as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 4, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M1: Complex Numbers and Transformations

Precalculus and Advanced Topics M2: Vectors and Matrices

Precalculus and Advanced Topics M4: Trigonometry

#### 5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Lessons in every module engage students in using appropriate tools strategically as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 5, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M2: Vectors and Matrices

Precalculus and Advanced Topics M4: Trigonometry

#### 6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions. Lessons in every module engage students in attending to precision as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 6, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M1: Complex Numbers and Transformations

Precalculus and Advanced Topics M2: Vectors and Matrices

Precalculus and Advanced Topics M3: Rational and Exponential Functions

#### 7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5-3(x-y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

Lessons in every module engage students in looking for and making use of structure as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 7, which is specifically addressed in the following modules:

Precalculus and Advanced Topics M3: Rational and Exponential Functions

# Aligned Components of Eureka Math

<b>8:</b> Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1), (x - 1)(x^2 + x + 1), and (x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Lessons in every module engage students in looking for and expressing regularity in repeated reasoning as required by this standard. This practice standard is analogous to the CCSSM Standards for Mathematical Practice 8, which is specifically addressed in the following modules: Precalculus and Advanced Topics M3: Rational and Exponential Functions Precalculus and Advanced Topics M5: Probability and Statistics
--	--

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
Number	Number	Cluster: Represent, interpret, co	mpare, and simplify number expressions.
and Quantity	Expressions	<b>P.N.NE.A.1</b> Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order	Algebra II M3 Topic A: Real Numbers Algebra II M3 Topic B: Logarithms
		to analyze them or compare them.	
		<b>P.N.NE.A.2</b> Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	Precalculus and Advanced Topics M3 Topic C: Inverse Functions
		<b>P.N.NE.A.3</b> Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of $\pi$ and <i>e</i> .	Algebra II M3 Lesson 16: Rational and Irrational Numbers

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.N.NE.A.4</b> Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not.	<ul> <li>Algebra II M1 Topic C: Solving and Applying Equations—Polynomial, Rational, and Radical</li> <li>Precalculus and Advanced Topics M3 Lesson 10: The Structure of Rational Expressions</li> <li>Precalculus and Advanced Topics M3 Lesson 11: Rational Functions</li> <li>Note: Supplemental material is necessary to completely address this standard.</li> </ul>
		<b>P.N.NE.A.5</b> Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	<ul> <li>Precalculus and Advanced Topics M3 Lesson 10: The Structure of Rational Expressions</li> <li>Precalculus and Advanced Topics M3 Lesson 11: Rational Functions</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
	The Complex Number	Cluster: Perform complex numb on the complex plane.	per arithmetic and understand the representation
	System	<b>P.N.CN.A.1</b> Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$ .	<ul> <li>Algebra II M1 Topic D: A Surprise from Geometry— Complex Numbers Overcome All Obstacles</li> <li>Precalculus and Advanced Topics M1 Lessons 7–8: Complex Number Division</li> </ul>
		<b>P.N.CN.A.2</b> Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	<ul> <li>Precalculus and Advanced Topics M1 Lessons 7–8: Complex Number Division</li> <li>Precalculus and Advanced Topics M1 Lesson 9: The Geometric Effect of Some Complex Arithmetic</li> <li>Precalculus and Advanced Topics M1 Lesson 17: The Geometric Effect of Multiplying by a Reciprocal</li> </ul>
		<b>P.N.CN.A.3</b> Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	Precalculus and Advanced Topics M1: Complex Numbers and Transformations

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.N.CN.A.4</b> Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.	<ul> <li>Precalculus and Advanced Topics M1 Lesson 6: Complex Numbers as Vectors</li> <li>Precalculus and Advanced Topics M1 Topic B: Complex Number Operations as Transformations</li> <li>Precalculus and Advanced Topics M1 Lessons 18–19: Exploiting the Connection to Trigonometry</li> <li>Precalculus and Advanced Topics M1 Lesson 20: Exploiting the Connection to Cartesian Coordinates</li> <li>Precalculus and Advanced Topics M2 Lesson 4: Linear Transformations Review</li> </ul>
		<b>P.N.CN.A.5</b> Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Precalculus and Advanced Topics M1 Lessons 11–12: Distance and Complex Numbers

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		Cluster: Use complex numbers in polynomial identities and equations.	
		<b>P.N.CN.B.6</b> Extend polynomial identities to the complex numbers.	<ul> <li>Precalculus and Advanced Topics M3 Lesson 1: Solutions to Polynomial Equations</li> <li>Precalculus and Advanced Topics M3 Lesson 2: Does Every Complex Number Have a Square Root?</li> <li>Precalculus and Advanced Topics M3 Lesson 3: Roots of Unity</li> </ul>
		<b>P.N.CN.B.</b> 7 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	<ul> <li>Precalculus and Advanced Topics M3 Lesson 1: Solutions to Polynomial Equations</li> <li>Precalculus and Advanced Topics M3 Lesson 2: Does Every Complex Number Have a Square Root?</li> <li>Precalculus and Advanced Topics M3 Lesson 3: Roots of Unity</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
	Vector and	Cluster: Represent and model wi	ith vector quantities.
	Matrix Quantities	<b>P.N.VM.A.1</b> Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$ , $ \boldsymbol{v} $ , $  \boldsymbol{v}  $ , $v$ ).	<ul> <li>Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane</li> <li>Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps</li> <li>Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges</li> </ul>
		<b>P.N.VM.A.2</b> Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Precalculus and Advanced Topics M2 Lesson 19: Directed Line Segments and Vectors Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges
		<b>P.N.VM.A.3</b> Solve problems involving velocity and other quantities that can be represented by vectors.	Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges Precalculus and Advanced Topics M2 Lesson 23: Why Are Vectors Useful?

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math	
		Cluster: Understand the graphic representation of vectors and vector arithmetic.		
		<b>P.N.VM.B.4</b> Add and subtract vectors.		
		a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	<ul> <li>Precalculus and Advanced Topics M2 Lesson 5: Coordinates of Points in Space</li> <li>Precalculus and Advanced Topics M2 Lesson 6: Linear Transformations as Matrices</li> <li>Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane</li> <li>Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps</li> <li>Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges</li> </ul>	
		b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	Precalculus and Advanced Topics M2 Lesson 20: Vectors and Stone Bridges Precalculus and Advanced Topics M2 Lesson 23: Why Are Vectors Useful?	

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$ , where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$ , with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps
		<b>P.N.VM.B.5</b> Multiply a vector by a scalar.	
		a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y).$	<ul> <li>Precalculus and Advanced Topics M2 Lesson 5: Coordinates of Points in Space</li> <li>Precalculus and Advanced Topics M2 Lesson 6: Linear Transformations as Matrices</li> <li>Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane</li> <li>Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content		Aligned Components of Eureka Math
		b. Compute the magnitude of a scalar multiple $cv$ using   cv   =  c v. Compute the direction of $cv$ knowing that when $ c v \neq 0$ , the direction of cv is either along $v$ (for $c > 0$ ) or against $v$ (for $c < 0$ ).		Precalculus and Advanced Topics M2 Lesson 17: Vectors in the Coordinate Plane Precalculus and Advanced Topics M2 Lesson 18: Vectors and Translation Maps
		<b>P.N.VM.B.6</b> Calculate and interpret the dot product of two vectors.		Precalculus and Advanced Topics M1 Lesson 21: The Hunt for Better Notation Precalculus and Advanced Topics M5 Lesson 7: Expected Value of a Discrete Random Variable
		Cluster: Perform operations on	ma	atrices and use matrices in applications.
		<b>P.N.VM.C.7</b> Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.		Precalculus and Advanced Topics M2 Topic A: Networks and Matrices

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.N.VM.C.8</b> Multiply matrices by scalars to	Precalculus and Advanced Topics M2 Topic A: Networks and Matrices
		produce new matrices, e.g., as when all of the payoffs in a game are doubled.	Precalculus and Advanced Topics M2 Lesson 4: Linear Transformations Review
			Precalculus and Advanced Topics M2 Lesson 5: Coordinates of Points in Space
			Precalculus and Advanced Topics M2 Lesson 6: Linear Transformations as Matrices
			Precalculus and Advanced Topics M2 Topic E: First- Person Video Games—Projection Matrices
		<b>P.N.VM.C.9</b> Add, subtract, and multiply	Precalculus and Advanced Topics M1 Lesson 22: Modeling Video Game Motion with Matrices
		matrices of appropriate dimensions.	Precalculus and Advanced Topics M1 Lesson 24: Matrix Notation Encompasses New Transformations!
			Precalculus and Advanced Topics M1 Lesson 25: Matrix Multiplication and Addition
			Precalculus and Advanced Topics M2: Vectors and Matrices

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.N.VM.C.10</b> Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	Precalculus and Advanced Topics M2 Lesson 10: Matrix Multiplication Is Not Commutative Precalculus and Advanced Topics M2 Lesson 12: Matrix Multiplication Is Distributive and Associative
		<b>P.N.VM.C.11</b> Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	<ul> <li>Precalculus and Advanced Topics M1 Topic C: The Power of the Right Notation</li> <li>Precalculus and Advanced Topics M2 Lesson 6: Linear Transformations as Matrices</li> <li>Precalculus and Advanced Topics M2 Lesson 13: Using Matrix Operations for Encryption</li> <li>Precalculus and Advanced Topics M2 Topic C: Systems of Linear Equations</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		P.N.VM.C.12 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	<ul> <li>Precalculus and Advanced Topics M1 Lesson 21: The Hunt for Better Notation</li> <li>Precalculus and Advanced Topics M1 Lessons 22–23: Modeling Video Game Motion with Matrices</li> <li>Precalculus and Advanced Topics M2 Lesson 4: Linear Transformations Review</li> <li>Precalculus and Advanced Topics M2 Lesson 7: Linear Transformations Applied to Cubes</li> <li>Precalculus and Advanced Topics M2 Lesson 11: Matrix Addition Is Commutative</li> <li>Precalculus and Advanced Topics M2 Topic D: Vectors in Plane and Space</li> <li>Precalculus and Advanced Topics M2 Topic E: First-Person Video Games—Projection Matrices</li> </ul>
		<b>P.N.VM.C.13</b> Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Precalculus and Advanced Topics M1 Topic C: The Power of the Right Notation Precalculus and Advanced Topics M2 Lesson 8: Composition of Linear Transformations

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
Algebra	Sequences	Cluster: Understand and use see	quences and series.
	and Series	<b>P.A.S.A.1</b> Demonstrate an understanding of sequences by representing them recursively and explicitly.	<ul> <li>Algebra I M3 Topic A: Linear and Exponential Sequences</li> <li>Algebra II M3 Lesson 25: Geometric Sequences and Exponential Growth and Decay</li> <li>Algebra II M3 Lesson 26: Percent Rate of Change</li> <li>Algebra II M3 Lesson 29: The Mathematics Behind a Structured Savings Plan</li> </ul>
		<b>P.A.S.A.2</b> Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.	Algebra II M3 Topic E: Geometric Series and Finance Note: Supplemental material is necessary to address infinite series.

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.A.S.A.3</b> Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist.	
		a. Determine whether a given arithmetic or geometric series converges or diverges.	Algebra II M3 Lesson 6: Euler's Number, <i>e</i> Algebra II M3 Lesson 26: Percent Rate of Change Note: Supplemental material is necessary to address converging or diverging arithmetic or geometric series.
		b. Find the sum of a given geometric series (both infinite and finite).	Algebra II M3 Topic E: Geometric Series and Finance Note: Supplemental material is necessary to address infinite geometric series.
		c. Find the sum of a finite arithmetic series.	Algebra II M3 Lesson 29: The Mathematics Behind a Structured Savings Plan

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.A.S.A.4</b> Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.	<i>Eureka Math</i> does not explicitly address this standard.
		<b>P.A.S.A.5</b> Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	Precalculus and Advanced Topics M3 Lessons 4–5: The Binomial Theorem
	Reasoning	Cluster: Solve systems of equation	ns and nonlinear inequalities.
	with Equations and Inequalities	<b>P.A.REI.A.1</b> Represent a system of linear equations as a single matrix equation in a vector variable.	Precalculus and Advanced Topics M2 Topic C: Systems of Linear Equations

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.A.REI.A.2</b> Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	Precalculus and Advanced Topics M2 Topic C: Systems of Linear Equations
		<b>P.A.REI.A.3</b> Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology.	<i>Eureka Math</i> does not explicitly address nonlinear inequalities.
		<b>P.A.REI.A.4</b> Solve systems of nonlinear inequalities by graphing.	<i>Eureka Math</i> does not explicitly address systems of nonlinear inequalities.

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
	Parametric	Cluster: Describe and use param	etric equations.
	Equations	<b>P.A.PE.A.1</b> Graph curves parametrically (by hand and with appropriate technology).	Precalculus and Advanced Topics M3 Lesson 6: Curves in the Complex Plane Precalculus and Advanced Topics M3 Lessons 7–8:
		<b>P.A.PE.A.2</b> Eliminate parameters by rewriting parametric equations as a single equation.	Curves from GeometryPrecalculus and Advanced Topics M2 Lesson 21: Vectors and the Equation of a LinePrecalculus and Advanced Topics M2 Lesson 22: Linear
			Transformations of Lines Precalculus and Advanced Topics M2 Lesson 24: Why Are Vectors Useful? Precalculus and Advanced Topics M3 Lesson 6: Curves
			in the Complex Plane Precalculus and Advanced Topics M3 Lessons 8: Curves from Geometry

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
	Conic Sections	Cluster: Understand the properti phenomena.	es of conic sections and model real-world
		<b>P.A.C.A.1</b> Display all of the conic sections as portions of a cone.	<ul> <li>Precalculus and Advanced Topics M3 Lesson 9: Volume and Cavalieri's Principle</li> <li>Note: Supplemental material is necessary to completely address this standard.</li> </ul>
		<b>P.A.C.A.2</b> Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Precalculus and Advanced Topics M3 Lesson 6: Curves in the Complex Plane Precalculus and Advanced Topics M3 Lessons 7–8: Curves from Geometry
		<b>P.A.C.A.3</b> From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics.	<ul> <li>Algebra I M4 Lesson 17: Graphing Quadratic Functions from the Standard Form, f(x) = ax<sup>2</sup> + bx + c</li> <li>Geometry M5 Topic D: Equations for Circles and Their Tangents</li> <li>Algebra II M1 Lesson 33: The Definition of a Parabola</li> <li>Precalculus and Advanced Topics M3 Lesson 6: Curves in the Complex Plane</li> <li>Precalculus and Advanced Topics M3 Lessons 7–8: Curves from Geometry</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		P.A.C.A.4 Transform equations of conic sections to convert between general and standard form.	Algebra I M4 Lesson 16: Graphing Quadratic Equations from the Vertex Form, $y = a(x - h)^2 + k$ Algebra I M4 Lesson 17: Graphing Quadratic Functions from the Standard Form, $f(x) = ax^2 + bx + c$ Geometry M5 Topic D: Equations for Circles and Their TangentsAlgebra II M1 Lesson 33: The Definition of a ParabolaPrecalculus and Advanced Topics M3 Lesson 6: Curves in the Complex Plane
			Precalculus and Advanced Topics M3 Lesson 8: Curves from Geometry
Functions	Building	Cluster: Build new functions from	n existing functions.
	Functions	P.F.BF.A.1	Algebra I M3 Topic C: Transformations of Functions
		Understand how the algebraic properties of an equation transform the geometric properties of its	Algebra II M2 Lesson 11: Transforming the Graph of the Sine Function
		graph.	Algebra II M3 Lesson 20: Transformations of the Graphs of Logarithmic and Exponential Functions
			Precalculus and Advanced Topics M3 Lesson 15: Transforming Rational Functions

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		c. Read values of an inverse function from a graph or a table, given that the function has an inverse.	Precalculus and Advanced Topics M3 Topic C: Inverse Functions
		d. Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.	Precalculus and Advanced Topics M3 Lesson 19: Restricting the Domain
		<b>P.F.BF.A.6</b> Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$ .	Precalculus and Advanced Topics M3 Topic C: Inverse Functions
	Interpreting Functions	Cluster: Analyze functions using	g different representations.
		<b>P.F.IF.A.1</b> Determine whether a function is even, odd, or neither.	<ul> <li>Algebra II M1 Lesson 15: Structure in Graphs of Polynomial Functions</li> <li>Precalculus and Advanced Topics M4 Lesson 2: Properties of Trigonometric Functions</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.BF.A.2</b> Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.	<ul> <li>Algebra II M2 Lesson 12: Ferris Wheels—Using Trigonometric Functions to Model Cyclical Behavior</li> <li>Algebra II M3 Lesson 28: Newton's Law of Cooling, Revisited</li> <li>Algebra II M3 Lesson 30: Buying a Car</li> <li>Algebra II M3 Lesson 33: The Million Dollar Problem</li> <li>Precalculus and Advanced Topics M3 Lesson 16: Function Composition</li> <li>Precalculus and Advanced Topics M3 Lesson 17: Solving Problems by Function Composition</li> <li>Precalculus and Advanced Topics M3 Topic C: Inverse Functions</li> </ul>
		<b>P.F.BF.A.3</b> Compose functions.	<ul> <li>Precalculus and Advanced Topics M3 Lesson 16:</li> <li>Function Composition</li> <li>Precalculus and Advanced Topics M3 Lesson 17:</li> <li>Solving Problems by Function Composition</li> </ul>
		<b>P.F.BF.A.4</b> Construct the difference quotient for a given function and simplify the resulting expression.	Algebra II M3 Lesson 6: Euler's Number, <i>e</i> Note: Supplemental material is necessary to address the difference quotient.

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.BF.A.5</b> Find inverse functions (including exponential, logarithmic, and trigonometric).	
		a. Calculate the inverse of a function, $f(x)$ , with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$ , the multiplicative inverse, 1/f(x), and the inverse with respect to composition, $f^{-1}(x)$ . Understand the algebraic and graphical implications of each type.	Precalculus and Advanced Topics M3 Topic C: Inverse Functions
		b. Verify by composition that one function is the inverse of another.	Precalculus and Advanced Topics M3 Topic C: Inverse Functions

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.IF.A.4</b> Identify the real zeros of a function and explain the relationship between the real zeros and the <i>x</i> -intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational).	<ul> <li>Algebra II M1 Lesson 11: The Special Role of Zero in Factoring</li> <li>Algebra II M1 Lesson 14: Graphing Factored Polynomials</li> <li>Algebra II M3 Lesson 17: Graphing the Logarithmic Function</li> <li>Precalculus and Advanced Topics M3 Lesson 14: Graphing Rational Functions</li> <li>Precalculus and Advanced Topics M4 Lesson 11: Revisiting the Graphs of the Trigonometric Functions</li> </ul>
		<b>P.F.IF.A.5</b> Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$ .	<ul> <li>Algebra II M2 Topic B: Understanding Trigonometric Functions and Putting Them to Use</li> <li>Algebra II M3 Lesson 20: Transformations of the Graphs of Logarithmic and Exponential Functions</li> <li>Precalculus and Advanced Topics M3 Lesson 15: Transforming Rational Functions</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.IF.A.2</b> Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).	<ul> <li>Algebra II M1 Lesson 14: Graphing Factored Polynomials</li> <li>Algebra II M1 Lesson 15: Structure in Graphs of Polynomial Functions</li> <li>Algebra II M1 Lesson 16: Modeling with Polynomials— An Introduction</li> <li>Algebra II M2 Lesson 12: Ferris Wheels—Using Trigonometric Functions to Model Cyclical Behavior</li> <li>Algebra II M2 Lesson 13: Tides, Sound Waves, and Stock Markets</li> <li>Algebra II M3 Lesson 33: The Million Dollar Problem</li> <li>Precalculus and Advanced Topics M3 Lesson 21: Logarithmic and Exponential Problem Solving</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<ul> <li>P.F.IF.A.6</li> <li>Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection.</li> <li>Describe intervals where the function is increasing or decreasing and where different types of concavity occur.</li> </ul>	<ul> <li>Algebra II M1 Lesson 14: Graphing Factored Polynomials</li> <li>Algebra II M1 Lesson 17: Modeling with Polynomials— An Introduction</li> <li>Note: Supplemental material is necessary to address points of inflection.</li> </ul>
		<b>P.F.IF.A.</b> <sub>7</sub> Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior.	Precalculus and Advanced Topics M3 Topic B: Rational Functions and Composition of Functions

Conceptual Category	Domain	Standards for Mathematical Content		Aligned Components of Eureka Math
		<b>P.F.IF.A.8</b> Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.		Algebra I M3 Lesson 2: Recursive Formulas for SequencesAlgebra I M3 Lesson 3: Arithmetic and Geometric SequencesAlgebra I M3 Lesson 4: Why Do Banks Pay YOU to Provide Their Services?Algebra II M3 Lesson 26: Percent Rate of Change
	Trigonometric Cluster: Extend the domain of trig Functions	rig	onometric functions using the unit circle.	
	1 unctions	<b>P.F.TF.A.1</b> Convert from radians to degrees and from degrees to radians.		Algebra II M2 Lesson 9: Awkward! Who Chose the Number 360, Anyway?

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.TF.A.4</b> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	<ul> <li>Algebra II M2 Lesson 11: Transforming the Graph of the Sine Function</li> <li>Algebra II M2 Lesson 12: Ferris Wheels—Using Trigonometric Functions to Model Cyclical Behavior</li> <li>Algebra II M2 Lesson 13: Tides, Sound Waves, and Stock Markets</li> <li>Precalculus and Advanced Topics M4 Lesson 6: Waves, Sinusoids, and Identities</li> </ul>
	Graphing	Cluster: Model periodic phenomer P.F.GT.A.1 Interpret transformations of trigonometric functions.	ena with trigonometric functions.
	Trigonometric Functions		Algebra II M2 Topic B: Understanding Trigonometric Functions and Putting Them to Use
		<b>P.F.GT.A.2</b> Determine the difference made by choice of units for angle measurement when graphing a trigonometric function.	Algebra II M2 Lesson 12: Ferris Wheels—Using Trigonometric Functions to Model Cyclical Behavior Algebra II M2 Lesson 13: Tides, Sound Waves, and Stock Markets

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.TF.A.2</b> Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ , and $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for $x$ , $\pi + x$ , and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number.	Algebra II M2 Lesson 4: From Circle-ometry to TrigonometryAlgebra II M2 Lesson 5: Extending the Domain of Sine and Cosine to All Real NumbersAlgebra II M2 Lesson 6: Why Call It Tangent?Algebra II M2 Lesson 10: Basic Trigonometric Identities from GraphsPrecalculus and Advanced Topics M4 Lesson 1: Special Triangles and the Unit Circle
		<b>P.F.TF.A.3</b> Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Precalculus and Advanced Topics M4 Lesson 2: Properties of Trigonometric Functions

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.GT.A.3</b> Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes.	Precalculus and Advanced Topics M4 Lesson 11: Revisiting the Graphs of the Trigonometric Functions
		<b>P.F.GT.A.4</b> Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions.	Precalculus and Advanced Topics M4 Topic C: Inverse Trigonometric Functions
		<b>P.F.GT.A.5</b> Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Precalculus and Advanced Topics M4 Lesson 12: Inverse Trigonometric Functions

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math	
Geometry	Applied	Cluster: Use trigonometry to solve problems.		
	Trigonometry	<b>P.G.AT.A.1</b> Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles.	Geometry M2 Topic E: Trigonometry	
		<b>P.G.AT.A.2</b> Derive the formula $A = 1/2$ <i>ab</i> sin( <i>C</i> ) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Geometry M2 Lesson 31: Using Trigonometry to Determine Area Precalculus and Advanced Topics M4 Lesson 7: An Area Formula for Triangles	
		<b>P.G.AT.A.3</b> Derive and apply the formulas for the area of sector of a circle.	Geometry M5 Topic B: Arcs and Sectors	

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.F.GT.A.6</b> Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions.	Precalculus and Advanced Topics M4 Topic C: Inverse Trigonometric Functions
		<b>P.F.GT.A.</b> <sub>7</sub> Graph the inverse trigonometric functions and identify their key characteristics.	Precalculus and Advanced Topics M4 Lesson 11: Revisiting the Graphs of the Trigonometric Functions
		<b>P.F.GT.A.8</b> Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	Precalculus and Advanced Topics M4 Topic C: Inverse Trigonometric Functions

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.G.AT.A.4</b> Calculate the arc length of a circle subtended by a central angle.	Geometry M5 Topic B: Arcs and Sectors
		<b>P.G.AT.A.5</b> Prove the Laws of Sines and Cosines and use them to solve problems.	Precalculus and Advanced Topics M4 Topic B: Trigonometry and Triangles
		<b>P.G.AT.A.6</b> Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	Geometry M2 Lesson 33: Applying the Laws of Sines and Cosines Precalculus and Advanced Topics M4 Lesson 10: Putting the Law of Cosines and the Law of Sines to Use
	Trigonometric		ntities to rewrite expressions and solve equations.
	Identities	<b>P.G.TI.A.1</b> Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle.	<ul> <li>Algebra II M2 Lesson 15: What Is a Trigonometric Identity?</li> <li>Algebra II M2 Lesson 16: Proving Trigonometric Identities</li> <li>Algebra II M2 Lesson 17: Trigonometric Identity Proofs</li> <li>Precalculus and Advanced Topics M4: Trigonometry</li> </ul>

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.G.TI.A.2</b> Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	Precalculus and Advanced Topics M4 Topic A: Trigonometric Functions
	Polar	Cluster: Use polar coordinates.	
	Coordinates	<b>P.G.PC.A.1</b> Graph functions in polar coordinates.	Precalculus and Advanced Topics M1 Lesson 13: Trigonometry and Complex Numbers
		<b>P.G.PC.A.2</b> Convert between rectangular and polar coordinates.	Precalculus and Advanced Topics M1 Lesson 13: Trigonometry and Complex Numbers Precalculus and Advanced Topics M1 Lessons 18–19: Exploiting the Connection to Trigonometry
		<b>P.G.PC.A.3</b> Represent situations and solve problems involving polar coordinates.	Precalculus and Advanced Topics M1 Lessons 18–19: Exploiting the Connection to Trigonometry

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math	
Statistics and Probability	Model with Data	Cluster: Model data using regressions equations.		
		<b>P.S.MD.A.1</b> Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real- world phenomena and to make predictions.	<ul> <li>Algebra I M2 Topic D: Numerical Data on Two Variables</li> <li>Algebra II M2 Lesson 13: Tides, Sound Waves, and Stock Markets</li> <li>Algebra II M3 Lesson 23: Bean Counting</li> <li>Algebra II M3 Lesson 27: Modeling with Exponential Functions</li> </ul>	
		<b>P.S.MD.A.2</b> Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data.	<ul> <li>Algebra I M2 Lessons 12–13: Relationships Between Two Numerical Variables</li> <li>Algebra I M2 Lesson 19: Interpreting Correlation</li> <li>Algebra I M2 Lesson 20: Analyzing Data Collected on Two Variables</li> <li>Algebra I M5 Lesson 7: Modeling a Context from Data</li> <li>Algebra II M2 Lesson 13: Tides, Sound Waves, and Stock Markets</li> <li>Algebra II M3 Lesson 27: Modeling with Exponential Functions</li> </ul>	

Conceptual Category	Domain	Standards for Mathematical Content	Aligned Components of Eureka Math
		<b>P.S.MD.A.3</b> Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.	<ul> <li>Algebra I M2 Lessons 12–13: Relationships Between Two Numerical Variables</li> <li>Algebra I M2 Lesson 19: Interpreting Correlation</li> <li>Algebra I M2 Lesson 20: Analyzing Data Collected on Two Variables</li> <li>Algebra I M5 Lesson 7: Modeling a Context from Data</li> <li>Algebra II M2 Lesson 13: Tides, Sound Waves, and Stock Markets</li> <li>Algebra II M3 Lesson 23: Bean Counting</li> <li>Algebra II M3 Lesson 27: Modeling with Exponential Functions</li> </ul>