

LEVELS K–5

**EUREKA  
MATH<sup>2</sup>**<sup>TM</sup>

# *Eureka Math*<sup>2</sup> <sup>TM</sup> Reviewer Rubric



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<b>Eureka Math<sup>2</sup> Grades K–5: Alignment at a Glance</b> <b>Criteria of High-Quality, Effective Math Programs</b>		Meets Criteria	
		Yes	No
<b>1. Content and Focus</b>			
A. Curricular materials align with standards and are mathematically accurate.		✓	
i. Sufficient time and resources are allocated to ensure the materials meet the full intent of grade-level standards.			
B. Curricular materials focus on major work of each grade level.		✓	
C. Curricular materials connect supporting and additional work to major work in meaningful ways.		✓	
i. Curricular materials make natural connections between different clusters and domains.			
<b>2. Coherence and Instructional Design</b>			
A. Curricular materials use a logical mathematical progression to build on learning from prior content.		✓	
B. Coherent high-yield models are evident within and across grade levels.		✓	
C. Curricular materials include an intentional sequence for developing academic and mathematical language within and across grade-level courses.		✓	
D. Curricular materials utilize a consistent module and lesson structure, including a variety of well-designed teacher-facilitated experiences.		✓	
E. Students' ideas are valued and seen as resources for learning.		✓	
F. Curricular materials build knowledge of not only key ideas in mathematics but also knowledge of the world.		✓	
G. Curricular materials are research-based.		✓	
<b>3. Balance of Rigor</b>			
A. Curricular materials contain a balance of conceptual understanding, procedural skills, and fluency, as well as application of math knowledge.		✓	
B. Curricular materials support the development of students' conceptual understanding with			
i. simple-to-complex problem sequences		✓	
ii. concrete-pictorial-symbolic progressions and connections			
C. Curricular materials are designed so that students attain the procedural skills and fluency required by grade-level standards.		✓	
D. Curricular materials are designed to include a variety of frequent, authentic application opportunities.		✓	
<b>4. Standards for Mathematical Practice</b>			
A. Curricular materials support the standards' emphasis on mathematical thinking and reasoning.		✓	

B. Curricular materials provide regular opportunities for students to engage in the full meaning of all the Standards for Mathematical Practice.	✓	
C. Curricular materials support teachers in developing their own understanding of the Standards for Mathematical Practice, the role of the practice standards in lessons, and guidance for implementation.	✓	
D. Curricular materials connect content standards and practice standards in authentic ways.	✓	
<b>5. Accessibility, Differentiation, and Engagement</b>		
A. Curricular materials intentionally promote student engagement, student-to-student discourse, and student ownership of learning.	✓	
B. Curricular materials ensure all students can access grade-level mathematics. <ul style="list-style-type: none"> <li>i. Lesson tasks provide multiple entry points into mathematics.</li> <li>ii. Curricular materials provide timely supports to assess and to address students' unfinished learning.</li> </ul>	✓	
C. Curricular materials are designed with principles of Universal Design for Learning by providing multiple means of engagement, representation, and action and expression.	✓	
D. Curricular materials provide scaffolds and instructional supports for multilingual learners.	✓	
E. Curricular materials provide differentiation suggestions for support and challenge.	✓	
F. Curricular materials attend to social and emotional learning (SEL) competencies: self-awareness, self-management, social awareness, relationship skills, and responsible decision-making.	✓	
G. Curricular materials are inclusive and reflect diverse cultures, ethnicities, and demographics.	✓	
H. Student facing-materials are visually engaging, accessible, and readable.	✓	
<b>6. Assessment Program</b>		
A. Curricular materials include frequent and varied and comprehensive assessments. <ul style="list-style-type: none"> <li>i. Summative assessments, taken as a whole, include opportunities for students to demonstrate the full intent of grade-level standards.</li> <li>ii. Formative assessments support teachers in determining whether students met the objective(s) of the lessons and topics.</li> <li>iii. Assessment items include a combination of tasks that require students to demonstrate conceptual understanding, procedural skill and fluency, and application.</li> <li>iv. Assessment item types require students to produce a variety of answers and solutions (arguments, explanations, representations, etc).</li> </ul>	✓	
B. Assessment materials provide sufficient guidance for interpreting student performance and guiding instructional decisions.	✓	
<b>7. Teachability and Digital Integration</b>		
A. Curricular materials include embedded and external professional development. <ul style="list-style-type: none"> <li>i. Materials support teacher learning and understanding of mathematical concepts, the progression of learning, and instructional pedagogy.</li> <li>ii. Materials include standards alignment information and explain the role of the standards in the resources.</li> </ul>	✓	
B. Curricular materials include support for implementation. <ul style="list-style-type: none"> <li>i. The materials are visually engaging, easy to use, and well organized for students and teachers.</li> <li>ii. Materials provide information about how to plan and prepare lessons.</li> </ul>	✓	

iii. Materials provide guidance for instructional delivery, including questions to prompt student thinking and expected student outcomes.		
C. Curricular materials can be completed within a regular school year, and guidance about the expected timing for lessons and tasks is provided.	✓	
D. Curricular materials provide caregivers with resources to support student academic progress.	✓	
E. A user-friendly online platform provides always-on access to curricular materials and additional resources.	✓	
F. Digital materials and experiences enhance classroom instructional practice, engaging students meaningfully to develop mathematical understanding.	✓	

Criteria of High-Quality, Effective Math Programs	Evidence of Alignment from <i>Eureka Math</i> <sup>2</sup> Levels K–5		
	Yes	No	Criteria Examples and Reviewer Notes
<b>1. Content and Focus</b>			
<p>A. Curricular materials align with standards and are mathematically accurate.</p> <p>i. Sufficient time and resources are allocated to ensure the materials meet the full intent of grade-level standards.</p>	✓		<p><i>Eureka Math</i><sup>2</sup> is designed to align fully with Common Core State Standards for Mathematics (CCSSM). The teacher–writers and mathematicians developed the curriculum by using the seminal <i>Progressions for the Common Core State Standards in Mathematics</i>, which lay out the structure of mathematics and research in cognitive development.</p> <p>The national curriculum attends to the CCSSM within and across lessons, topics, and modules. The <i>Teach</i> books contain resources for identifying the standards addressed at the lesson and module level. For example, the Standards resource at the end of each module’s <i>Teach</i> book lists the standards addressed in each module. Standards are also listed on the first page of each lesson.</p> <p>A universal edition (without CCSSM references and alignments), specific state editions, and state alignment guides (demonstrate how each level of <i>Eureka Math</i><sup>2</sup> aligns with specific state standards) are in development.</p>
<p>B. Curricular materials focus on major work of each grade level.</p>	✓		<p>The <i>Eureka Math</i><sup>2</sup> curriculum prioritizes major work standards by devoting more time to those standards overall and by sequencing modules to teach the major work as early in the school year as possible. Such standards appear early and build over the course of the year in accordance with the progressions in the standards so that students have ample opportunity to work with them.</p> <p>For example, in the level 1 curriculum, Units of Ten, all six modules focus on major work of level 1: addition and subtraction, place value, and measurement. This major work is explicitly taught and applied in 118 out of 140 lessons (84%). In these 118 lessons, the majority of class time (50 or more minutes of 60 minutes) is focused on major work. Of the 140 lessons, 12% of the lessons are aligned with additional content standards and 5% of the lessons are aligned with supporting content standards.</p> <p>Refer to the module map, Appendix A.</p>
<p>C. Curricular materials connect supporting and additional work to major work in meaningful ways.</p> <p>i. Curricular materials make</p>	✓		<p><i>Eureka Math</i><sup>2</sup> lessons and materials make natural connections between different clusters and domains. Whenever possible, the major work of the level is naturally embedded in lessons even when the objective targets supporting or additional work. This natural embedding supports students in making meaningful content connections so that they understand mathematics as a coherent discipline, not as a series of unrelated topics.</p> <p>For example, the objectives in module 2 of kindergarten target additional work that appears in the Geometry domain: identifying, describing, and composing two- and three-dimensional shapes. However, composing shapes also prepares students for major work that appears in the Operations and Algebraic Thinking domain: making sense of part–part–whole relationships to decompose numbers in more than one way.</p> <p>Curriculum materials also intentionally make important connections among two or more clusters in a domain. For example, in level 1, the concept of</p>

<p>natural connections between different clusters and domains.</p>		<p>equality found in the Operations and Algebraic Thinking domain is carefully woven into each module, connecting this major work to the other domains:</p> <ul style="list-style-type: none"> <li>• In the Measurement and Data domain, students compare lengths or categories and make them equal. <ul style="list-style-type: none"> <li>○ To develop the concept of place value where 10 ones = 1 ten, students use 10-centimeter rods and 1-centimeter cubes rather than using only 1-centimeter cubes to measure lengths.</li> </ul> </li> <li>• In the Numbers and Operations in Base Ten domain, while exploring ways to make easier addition problems, students create expressions that are easier yet equivalent to the original expression.</li> <li>• Students also connect equality to composing and decomposing two-dimensional and three-dimensional shapes in the Geometry domain.</li> </ul>
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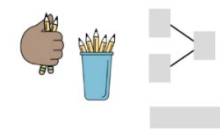
## 2. Coherence and Instructional Design

<p>A. Curricular materials use a logical mathematical progression to build on learning from prior content.</p>	<p>✓</p>	<p>The <i>Eureka Math</i><sup>2</sup> teacher–writers carefully constructed the curriculum as a logical mathematical <i>Story of Units</i>. Rather than separate disjointed skills, the concepts within modules and across levels are intentionally coherent and connected to the larger progression of mathematical concepts over time.</p> <p>Within each lesson, problems and exercises follow an intentional sequence that gradually reduces supports, which promotes student discovery and productive struggle. Through this process, students apply previous knowledge to new learning. A layered approach directs teachers to strategically revisit skills in increasingly complex ways so that students develop proficiency gradually over time.</p> <p>Teacher–writers intentionally sequenced new learning to best build on students’ conceptual understanding from previous learning. Embedded professional development helps teachers understand these choices and realize how the content builds on itself.</p> <p>For example, the Why section of the level 5 module 1 Overview answers the question Why does multiplication and division of whole numbers come first? The section explains why the choice was made to introduce this concept first and how teachers can help students build on this learning throughout the school year.</p> <p>Throughout the modules, teachers will find explicit references to learning from previous grade levels. For example, Module Overviews in the <i>Teach</i> book contain Before This Module and After This Module features, which connect the module’s new learning to that of previous and future modules and grade levels, revealing the coherent structure of the curriculum.</p> <p>For example, in level 2 module 5, students use place value strategies and the properties of operations to solve problems with coins and bills and to measure and estimate lengths to solve measurement problems. The Before This Module section informs teachers about the work students have already done in level 2 module 1 and module 4 with measurement, place value representations, and properties of operations. Then, in the After This Module section, the teacher learns how students will apply the knowledge they build in this module to new learning in level 3 module 5, when they partition lengths to represent fractions on the number line, and in level 3 module 6, when they define perimeter and study two-dimensional shapes.</p>
<p>B. Coherent high-leverage models are evident within and across grade levels.</p>	<p>✓</p>	<p><i>Eureka Math</i><sup>2</sup> uses a core set of coherent physical and pictorial models within and across grade levels. Many models used in levels K–5 evolve along with the growing complexities of mathematics and are representative of similar models that appear in levels 6–12. These models activate students’ memory and emphasizes how the same math concepts can apply to new, more complex units. Here are some examples of the coherence of models:</p> <ul style="list-style-type: none"> <li>• Ten frames and 10-group arrays evolve into arrays of varying units. In later levels, students use a related area model for multiplication.</li> <li>• Hands and the number path are early linear models used to represent whole numbers. These models evolve into the use of number lines to represent a larger variety of number types, including fractions and decimals.</li> </ul>

- Place value disks and place value charts are used starting in level 2 and through level 5 to model numbers and to compute with understanding.
- Tape diagrams are taught in level 1 and used well beyond level 5 to represent word problems and discover a solution path.
- Various arrays, such as 5-groups and area models, are used from kindergarten through level 5 and beyond to support efficient counting and computing with conceptual understanding.
- Number bonds are used throughout the curriculum to model part–whole relationships of various units and for simplifying strategies for addition, subtraction, multiplication, and division.

To take one of these examples, consider how the number bond model is a high-leverage model that is used throughout the curriculum:

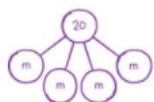
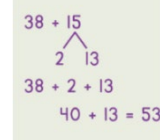
In kindergarten, students use the number bond to understand part–part–total relationships and to represent math stories such as this: The student has two pencils, and there are five pencils in the cup, so there are seven pencils altogether.



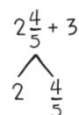
In level 1, students consider the part–whole–relationships represented by number bonds to find an unknown in a word problem or equation. For example, to solve  $9 - 3 = ?$ , students may think, “I know 3 and 6 make 9, so the missing part is 6.”



In levels 1 and up, students use number bonds to show how they used decomposition and/or composition to make an easier problem.



In level 3, students may also use a number bond to compose and decompose a total as two or more equal groups.

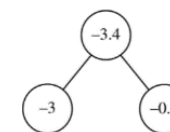


By level 5, students may use number bonds to compose and decompose fractions and decimals.

$$2 + 3 = 5$$

$$5 + \frac{4}{5} = 5\frac{4}{5}$$

Later in level 7, students use the number bond to compose and decompose rational numbers.



C. Curricular materials include an intentional sequence for developing academic and mathematical language within

✓

*Eureka Math*<sup>2</sup> uses accurate terminology that remains consistent across grade levels. This ensures that younger students are prepared for later grade levels and will recognize concepts they have already learned.

The Terminology resource in the *Teach* book of each module lists the specialized language of mathematics that is used throughout a module. The resource includes new and familiar terminology with definitions and descriptions from the module. Items in the New category are domain-specific words that are introduced to students in the module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes language for teachers that expands how the words are used with students. This extra guidance is set in italics and follows the term it refers to. Items in the Familiar category are domain-specific words introduced in prior modules or grade levels.



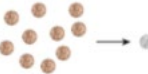
Terminology is systematically introduced and developed within lessons by using student-friendly descriptions and definitions. Lessons generally

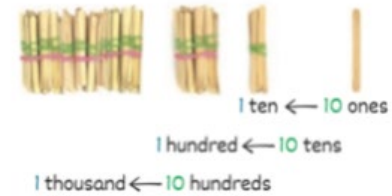


<p>and across grade-level courses.</p>		<p>create experiences with the relevant concepts or ideas first before introducing a specific term. New terms are formalized through sample dialogue that models use of the term and its definition or description. When new terms are introduced, instructional recommendations are included either in the teacher guidance of the lesson or in a language support margin note. Examples of these recommendations include constructing or co-constructing a graphic organizer, engaging in a visual representation, or authentically using the word. Students formalize terminology after those experiences so that the new language comes as students need it to describe, reflect on, or name their work. After developing understanding of domain-specific terminology, students are encouraged to attend to precision by using vocabulary correctly.</p> <p>Academic verbs are also identified in the Terminology resource of the module in which they are introduced. The introduction of academic verbs such as <i>defend</i>, <i>classify</i>, and <i>consider</i> are intentionally distributed and sequenced across grade levels. When introduced, academic verbs are always paired with student-friendly language to support understanding.</p> <p>The last sentence of the Lesson at a Glance states whether any terminology is introduced in the lesson. New domain-specific terminology is also made bold in the lesson text when it is introduced. Often, terminology has accompanying Language Support notes. Students and caretakers can find new terminology and definitions in the Practice Partner for each relevant lesson in the student's <i>Apply</i> book.</p>
<p>D. Curricular materials utilize a consistent module and lesson structure, including a variety of well-designed teacher-facilitated experiences.</p>	<p>✓</p>	<p>Well-designed teacher-facilitated experiences that vary in setting and tempo have the highest capacity for equitably building new knowledge and should therefore be the primary mode of instruction. These experiences should include periods of whole-class engagement, small-group and paired collaborations, and independent reflection and processing to accommodate a variety of learning needs. Facilitation styles and lesson experiences also vary—explicit instruction, inquiry, guided discovery, etc.</p> <p><i>Eureka Math</i><sup>2</sup> maintains a consistent structure and design across all grade levels. Each level is composed of six modules, with each module further divided into topics. Inside topics, each lesson consists of the same four components: Fluency, Launch, Learn, and Land.</p> <p>Levels K–5 include a variety of engaging fluency activities with each lesson. The first component, Fluency, provides a combination of interleaved and distributed practice to support students in achieving fluency expectations. Fluency activities are designed to prepare students for new learning by activating prior knowledge, extending practice for proficiency, and bridging small learning gaps.</p> <p>The second component, Launch, creates an accessible entry point to the day's learning. Low-floor/high-ceiling activities build context and often create productive struggle that leads to a need for the learning that follows. Launch closes with a transition statement that makes the learning objective clear to all students. In alignment with Universal Design for Learning guidelines, the transition statement is written in student-friendly language and conveys the goals of the lesson so students can self-monitor their progress toward meeting the lesson objective.</p> <p>The next component, Learn, presents new learning related to the lesson objective, usually through a series of instructional segments. This lesson component fills most of the instructional time. Suggested facilitation styles vary and may include direct instruction, guided instruction, group work, partner activities, interactive video, or digital elements. The Learn component also includes a Problem Set, which serves as an opportunity for differentiated independent practice for students.</p> <p>The final component, Land, is a brief discussion to close the lesson and provides students with an opportunity to complete the Exit Ticket. Suggested questions, including Key Questions related to the objective, help students synthesize the day's learning. The Exit Ticket provides a window into what students understand to inform the teacher's instructional decisions about what they need next.</p>
<p>E. Students' ideas are valued and seen as resources for learning.</p>	<p>✓</p>	<p>Math is a social activity, and everyone benefits from learning interactively about mathematics and from one another's thinking. Lessons support a growth mindset ("I/you can learn mathematics."), provide opportunities for students to engage in productive struggle, and help them feel they belong in the larger community of student-mathematicians. Students come to see that they are capable in mathematics and that what they contribute is valuable to the math community they are a part of.</p> <p>Through rich, relevant, and rigorous tasks and opportunities for discourse, students and educators deepen their understanding of both the mathematics and one another. The math writing team used a wide research-base, including Mary Kay Stein and Peg Smith's <i>5 Practices for</i></p>

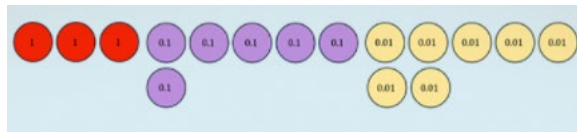
		<p><i>Orchestrating Productive Mathematics Discussions</i> to support teachers in engaging students with these tasks and conversations.</p> <p>Every lesson offers classroom discussions, partner or group talk, and rich questions to promote student discourse and elevate student metacognition. Many lessons highlight and provide opportunities for students to generate and share multiple solution pathways and representations. The lesson provides support through suggested routines, sample responses, discussion questions and more to guide teachers through the <i>5 Practices</i> framework to anticipate, monitor, select, sequence, and connect student responses.</p>
F. Curricular materials build knowledge of not only key ideas in mathematics but also knowledge of the world.	✓	<p><i>Eureka Math</i><sup>2</sup> builds mathematical knowledge in the context of real-world situations. As such, <i>Eureka Math</i><sup>2</sup> incorporates other disciplines, such as fine art and history, that encourage students explore what math can teach about the world around them.</p> <p>Each module integrates a stunning work of fine art that has a connection to the math learning in the module. The cover art and other carefully curated pieces of art and artists are discussed or analyzed in at least one lesson per module.</p> <p>In addition, each module contains a Math Past resource. Math Past tells the history of a big idea that has shaped the mathematics introduced in the module. The content in each Math Past highlights contributions from cultures around the world. It presents students with history that frames mathematics exploration as a human endeavor by telling the story of the discipline through artifacts and discoveries.</p> <p>Wordless context videos also serve as an engaging format for students to make observations and develop questions. These videos appear frequently and provide access to a mathematical context. Students use the context for their own mathematical wonderings and to generate discourse as they create their own word problems and answer their own questions.</p>
G. Curricular materials are research-based.	✓	<p>Experienced and qualified teacher–writers applied eight years of <i>Eureka Math</i> classroom experiences, student data, and current educational research to create <i>Eureka Math</i><sup>2</sup>. Highly educated mathematicians collaborated during the architecture phase of writing and performed thorough reviews of all curricular materials. Writing editors and math auditors also reviewed all materials.</p> <p>This thorough research informed the instructional design of the curriculum, guiding decisions in each lesson, topic, and module to combine best practices in pedagogy with rigorous math content. The research base includes the <i>Progressions for the Common Core State Standards in Mathematics</i> and texts related to mathematics and mathematical strategies, pedagogy and teaching philosophy, history of mathematics, instructional strategies, and coherence among the mathematical concepts. At the end of each module’s <i>Teach</i> book, a <i>Works Cited</i> section outlines the robust knowledge base that underpins the structure and content framework.</p> <p>See Selection of <i>Eureka Math</i><sup>2</sup> Research Citations in Appendix B.</p>
<b>3. Balance of Rigor</b>		
A. Curricular materials contain a balance of conceptual understanding, procedural skills and fluency, as well as application of math knowledge.	✓	<p><i>Eureka Math</i><sup>2</sup> balances the three elements of rigorous mathematics education: conceptual understanding, procedural skill and fluency, and application to real-world contexts. Lessons provide experiences to expand, develop, apply, and practice conceptual understanding, as well as hone procedural and problem-solving skills through joyful fluency exercises and engaging real-world applications.</p> <p>The curriculum sometimes presents the three elements separately but also often combines them, reinforcing the development of students as flexible thinkers on the path to proficiency <i>Eureka Math</i><sup>2</sup> addresses these areas with equal intensity, recognizing that all are vital parts of a coherent, effective curriculum.</p> <ul style="list-style-type: none"> <li>• The Fluency component provides daily opportunities for students to practice concepts and procedural skills.</li> <li>• Activities in Launch and Learn generally include a mix of the three elements of rigorous mathematics education. However, Launch often provides an application opportunity for students to engage in some contextual reasoning and problem solving.</li> <li>• Each segment of Learn, the instructional component of the lesson, often focuses on developing conceptual understanding and is</li> </ul>

		<p>structured to move students from simple problems to more complex problems.</p> <ul style="list-style-type: none"> <li>Land provides an opportunity to consolidate student understanding and reflect on their learning through discussion, and it highlights the Key Questions of the lesson.</li> </ul> <p>All three elements of rigor are also present in the Practice problems and assessments. For example, in level 5 module 3 lesson 4, students are given a set of multiplication problems (procedural skill and fluency), several word problems (application), and an opportunity to explain their thinking (conceptual understanding).</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="border: 1px solid black; padding: 5px; width: 150px;"> <p>Multiply.</p> <p>9. <math>\frac{1}{11} \times 8 = \underline{\hspace{2cm}}</math></p> </div> <div style="border: 1px solid black; padding: 5px; width: 350px;"> <p>Use the Read–Draw–Write process to solve each problem.</p> <p>17. Mr. Evans makes 10 pints of salsa. <math>\frac{3}{4}</math> of the pints of salsa are spicy. How many pints of the salsa are spicy?</p> </div> </div> <div style="border: 1px solid black; padding: 5px; width: 350px; margin-top: 10px;"> <p>3. Which expression results in a product greater than 4? Explain how you know.</p> <p style="text-align: center;"><math>\frac{3}{4} \times 4</math>      <math>\frac{5}{4} \times 4</math></p> </div>
<p>B. Curricular materials support the development of students’ conceptual understanding with</p> <ul style="list-style-type: none"> <li>i. simple–to–complex problem sequences</li> <li>ii. concrete–pictorial–symbolic progressions and connections</li> </ul>	<p>✓</p>	<p><i>Eureka Math</i><sup>2</sup> develops students’ conceptual understanding in several critical ways, emphasizing deep knowledge building and ensuring that students understand the “why” rather than just the “how” of math. Concepts progress coherently within and across modules and levels, so students build on what they already know. Lessons invite students to engage and respond with a variety of representations: visual, symbolic, verbal, contextual, and physical.</p> <p>Throughout <i>Eureka Math</i><sup>2</sup>, concepts are developed with a concrete–pictorial–abstract sequence. Students first explore concepts through the hands-on use of manipulatives, or concrete models. The concrete stage is vital to conceptual understanding and is maintained throughout the elementary curriculum when new concepts are introduced. Even when the lessons have progressed to more sophisticated representations, their use is encouraged for students when needed as support or as a representation choice.</p> <p>Students then progress to pictorial models that are connected to the related concrete model, when applicable. These models are often more convenient, further develop student understanding, and expand students’ problem-solving toolbox. Lessons support teachers in connecting these pictorial models to their corresponding abstract or symbolic representations, highlighting connections between the two. At the end of the progression, as students are ready, they may need only an abstract representation (or even a visual/mental model). This progression builds a strong foundation of number sense and provides students with a deep conceptual understanding.</p> <p>Consider this place value example:</p> <p>In kindergarten through level 2, students use concrete models to represent making the next higher unit by grouping, bundling, or trading. They will write the corresponding abstract numeral to label the representation.</p>

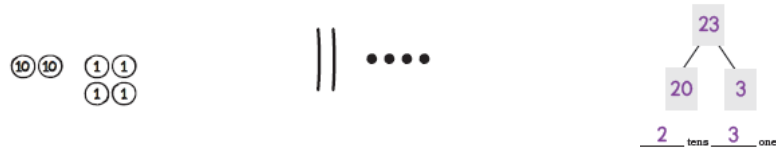
Groupable	Pre-grouped	Nonproportional
<p>Physically group 10 ones to compose a new unit of ten. The size of the new unit is proportionally larger than the base unit.</p> 	<p>Tens are composed by trading 10 ones for a new item that represents 1 ten. The new item is proportionally larger than the base unit.</p> 	<p>10 ones are traded for a new item that represents 1 ten. However, the new item is non-proportional. Visually, it is not 10 times larger than the base unit.</p> 



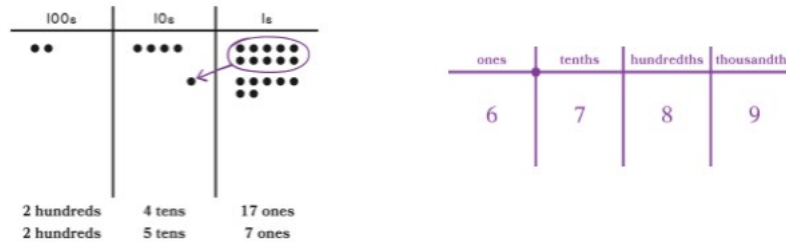
In levels 2–5, students represent the value of digits in a written numeral by using place value disks.



They may draw to show the place value or use number bonds to represent the place value of numbers.



They also represent numbers by drawing or writing on place value charts.



Students in all levels are encouraged to use various written representations of numbers, including unit form.

$$6.789$$

$$6 \frac{789}{1,000}$$

6 ones 7 tenths 8 hundredths 9 thousandths  
six and seven hundred eighty-nine thousandths

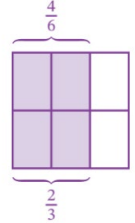
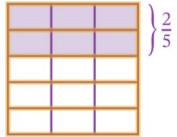
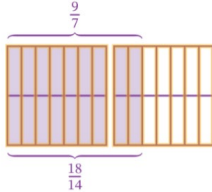
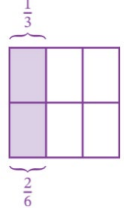
$$4 \times 10 + 2 \times 1 + 8 \times 0.1 + 5 \times 0.01 + 3 \times 0.001$$


$$4 \times 10 + 2 \times 1 + 8 \times \frac{1}{10} + 5 \times \frac{1}{100} + 3 \times \frac{1}{1,000} \quad 57 \times 10^3$$

When solving word problems, students are encouraged to use both pictorial and abstract representations. They may self-select concrete tools.

Simple-to-complex problem sequences in lessons and problem sets also support students' development of conceptual understanding by incrementally introducing new complexities, building on previous knowledge.

For example, these problems progress from simple to complex and include the complexities of addition and subtraction, renaming fractions with smaller units, renaming fractions with larger units, renaming fractions less than one, and renaming fractions greater than one.

$\frac{7}{6} - \frac{2}{3}$	 $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ $\frac{7}{6} - \frac{2}{3} = \frac{7}{6} - \frac{4}{6} = \frac{3}{6} = \frac{1}{2}$
$\frac{6}{15} + \frac{4}{5}$	 $\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$ $\frac{6}{15} + \frac{4}{5} = \frac{2}{5} + \frac{4}{5} = \frac{6}{5}$
$1 + \frac{18}{14}$	 $\frac{18}{14} = \frac{18 \div 2}{14 \div 2} = \frac{9}{7}$ $1 + \frac{18}{14} = 1 + \frac{9}{7} = \frac{16}{7}$
$\frac{15}{6} - \frac{1}{3}$	 $\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ $\frac{15}{6} - \frac{1}{3} = \frac{15}{6} - \frac{2}{6} = \frac{13}{6}$

<p>C. Curricular materials are designed so that students attain the procedural skills and fluency required by grade-level standards.</p>	<p>✓</p>	<p>Each <i>Eureka Math</i><sup>2</sup> lesson begins with a short series of engaging fluency activities. Students actively participate in their own learning during fluency exercises. Whole group activities are carefully designed to promote and celebrate individual growth while stimulating the joy in mathematics. The curriculum features a wide variety of fluency activities. One purpose of these activities is to help students practice and strengthen previously learned skills. Teachers can also use these activities as a check for understanding, identifying opportunities for intervention. Fluencies may be customized by teachers to meet their students' needs.</p> <p>Fluency with procedural skills is expected only after students have had time to develop conceptual understanding. In addition to the Fluency segment of each lesson, procedural skills and fluency are achieved through a variety of activities embedded in the materials.</p> <ul style="list-style-type: none"> <li>• Lessons include explicit instruction with conceptual understanding to achieve proficiency in procedural skills and to develop strategies for fluency of facts.</li> <li>• Each lesson includes a set of practice problems, organized from simple to complex, targeting the lesson objective(s). This Problem Set provides students with ample independent practice and opportunities for teacher feedback.</li> <li>• In the <i>Apply</i> student book, the Practice includes Remember problems that distribute and interleave previously learned concepts and skills. At a minimum, each key concept returns 10, 30, 60, and 100 days after instruction. These problems provide skills practice that intentionally builds towards grade-level fluency expectations and anticipates upcoming skills or maintains previously learned grade-level skills.</li> </ul>
<p>D. Curricular materials are designed to include a variety of frequent authentic application opportunities.</p>	<p>✓</p>	<p>Many lessons provide students with application experiences that move math off the page and into the world around them. For example, real-world contexts are used throughout each module both to motivate the learning of a new concept or skill and to provide opportunities to apply new learning. For example, students in level 1 consider a context of having some pennies and finding how many they need to get a certain total. In levels 1, 3, and 5, students are presented with scenarios about sharing food in various ways to solve division and fraction problems.</p> <p>Wordless context videos present a real-world mathematical situation and create student-driven problem solving. This format provides opportunities for authentic problem-solving and helps students understand unfamiliar contexts.</p> <p>These images, from videos in levels 3 and 4, for example, portray a factory employee using mathematics to calculate the day's production and friends determining rollercoaster car capacity at an amusement park.</p>  <p>A variety of word problem types and complexities are used throughout the curriculum to provide students with ample opportunity to apply new learning. For example, in kindergarten through level 2, students get experience with all addition and subtraction word problem types—<i>add to</i>, <i>take from</i>, <i>put together/take apart</i>, and <i>compare</i>. (See chart below.) Then in level 3, students continue to work with the same word problem types with one or two steps and larger numbers. They also work with all the multiplication and division word problem types—<i>equal groups</i>, <i>arrays</i>, <i>area</i>, <i>compare</i>, and <i>general</i>. Students continue to apply their learning in levels 4 and 5 as they work with all problem types and solve one- or two-step problems that contain different types of numbers, including fractions.</p>

	Result Unknown	Change Unknown	Start Unknown
Add to	Val gets 8 points. Then she gets 7 points. How many points does she have?  $8 + 7 = ?$	Jade has 12 tickets. She gets some more tickets. Now she has 20 tickets. How many tickets did she get?  $12 + ? = 20$ or $20 - 12 = ?$	Kit has some dollars. She gets 7 more dollars. Now she has 15 dollars. How many dollars did she start with?  $? + 7 = 15$ or $15 - 7 = ?$
Take From	Baz has 20 points. He uses 19 points. How many points does he have left?  $19 + ? = 20$ or $20 - 19 = ?$	Zan has 15 points. He uses some points. Now he has 8 points left. How many points did he use?  $? + 8 = 15$ or $15 - 8 = ?$	Baz has some dollars. He spends 10 dollars. He has 2 dollars left. How many dollars did he have at first?  $? - 10 = 2$ or $10 + 2 = ?$
Put Together/ Take Apart	<b>Total Unknown</b> Max has 12 tickets. Kat has 8 tickets. How many tickets do they have in all?  $12 + 8 = ?$	<b>Addends Unknown</b> Val hit 5 balloons. She got 19 points. What points were on the balloons?  $19 = ? + ? + ?$	<b>Addend Unknown</b> Tam has 9 points. Baz has some points. They have 15 points in all. How many points does Baz have?  $9 + ? = 15$ or $15 - 9 = ?$
Compare	<b>Difference Unknown</b> Tam plays for 12 minutes. Max plays for 4 minutes. How many more minutes does Tam play than Max plays?  $4 + ? = 12$ or $12 - ? = 4$	<b>Bigger/Longer Unknown</b> Jon plays for 8 minutes. Deb plays for 2 more minutes than Jon plays. How long does Deb play?  $8 + ? = ?$	<b>Smaller/Shorter Unknown</b> Kit plays for 10 minutes. Ben plays for 4 fewer minutes than Kit plays. How long does Ben play?  $10 - 4 = ?$ or $? + 4 = 10$
	Math takes 12 minutes. Art takes 4 minutes. How many fewer minutes does art take than math takes?  $4 + ? = 12$ or $12 - ? = 4$	(shorter suggests wrong operation) Val's hand is 6 cubes long. Val's hand is 2 cubes shorter than Mom's hand. How long is Mom's hand?  $6 + 2 = ?$	(longer suggests wrong operation) Val's hand is 6 cubes long. Val's hand is 2 cubes longer than her friend's hand. How long is her friend's hand?  $6 - 2 = ?$

The Read–Draw–Write (RDW) process supports students as they make sense of word problems, choose and apply mathematics, and solve:

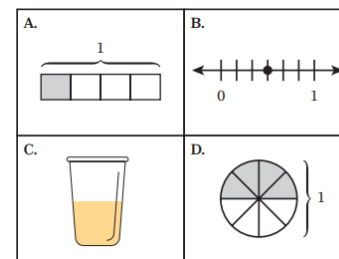
Read the problem all the way through. Then reread a chunk at a time. As you reread, ask yourself, “Can I draw something?” Then ask, “What can I draw?”

Draw to represent the problem as you reread. Add to or revise your drawing as you uncover new information or discover what is unknown. As you draw, label what is known and what is unknown. When you finish rereading and drawing, ask yourself, “What does my drawing show me?” Let your drawing help you find a way to solve.

Write number sentences or equations to represent your thinking. Solve.

This supports the levels 6–8 Read–Represent–Solve–Summarize process and leads to the modeling cycle in Algebra I.

Additionally, students often apply reasoning and mathematical thinking in open-middle and open-ended tasks in lessons. These tasks push students to ask questions, create models to represent real-world problems, and solve problems using self-selected tools and strategies. For example, using the Which One Doesn't Belong instructional routine, students identify a category in which three of the pictures belong, but a fourth does not. These problems are designed to have reasonable justifications why any of the pictures don't belong, and teachers facilitate a conversation about the different ways of viewing the problem. In this example from level 4, students might say, for example, that A does not belong because it is the only option that represents  $\frac{1}{4}$  while the others represent  $\frac{1}{2}$ . Or students might say that C does not belong because it is the only one that is an estimate, while the others are partitioned precisely.



The Thinking Tool is an additional resource to guide student critical thinking and problem-solving strategies. The Thinking Tool is a scaffold to support students in developing and applying critical thinking and metacognitive skills. It provides a set of questions students can ask themselves before, during, and after engaging in a task, encouraging them to analyze their own thinking processes and strategies in real-time. Furthermore, the After prompts in the Thinking Tool challenge students to set goals for revising or refining their strategies and processes next time. Students refer to the Thinking Tool as the teacher models self-talk and self-questioning as part of a think-aloud.



#### 4. Standards for Mathematical Practice

A. Curricular materials support the standards' emphasis on mathematical thinking and reasoning.

✓

*Eureka Math*<sup>2</sup> prioritizes attention to the Standards for Mathematical Practice (MPs) over the course of each year, addressing all eight MPs in each module. The MPs are the practices necessary for students to reason mathematically, communicate conceptual understanding, and represent and solve problems. Rich tasks, development of flexible thinking, and frequent opportunities for student discourse encourage students to engage with the MPs during all lesson components.

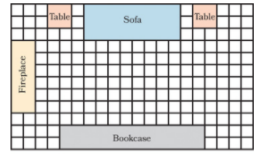
The Standards for Mathematical Practice (MPs) are intentionally woven into each lesson. Through explicit instruction and student-driven tasks, students consistently apply MPs to their daily work to build an enduring understanding of math. Curricular materials support the standards' emphasis on mathematical thinking and reasoning.

Materials were intentionally designed to identify each practice standard in all six modules of each level or course. Every lesson includes at least one Standards for Mathematical Practice margin note that highlights an opportunity to promote the MPs and identifies how the activities or problems in the lesson meet the practice standard. In addition to citing an aligned practice, the callout provides student prompts that help teachers promote the practice. Because of the interconnected nature of the Mathematical Practices, engagement with one MP often leads to engagement with others. For example, students who are reasoning about the quantities in a problem (MP2) need to understand the meaning of the problem and the relationship of those quantities (MP1).

Though a lesson typically engages students in multiple practices, only the one with the strongest alignment with lesson content is noted and promoted. Collectively, the instances in a level or course identifying student engagement in a particular math practice attend to the full meaning of the practice standard.




<p>B. Curricular materials provide regular opportunities for students to engage in the full meaning of all the Standards for Mathematical Practice.</p>	<p>✓</p>	<p>The Promoting the Standards for Mathematical Practice margin notes in each lesson were intentionally crafted to attend to the full meaning of the practice standards and to build the habits of mind, such as problem solving, reasoning, and modeling, that the MP intends to shape or support.</p> <p>The bulleted questions in each MP margin note promote engagement in the practice, are written in grade-level appropriate language, and are coherent across grade levels.</p> <p>The following set of margin notes illustrate the coherence of MP3 across grade levels, which requires students to construct viable arguments and to critique the reasoning of others.</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="558 350 947 719" style="border: 1px solid #ccc; padding: 10px; background-color: #e6f2ff;"> <p><b>Promoting the Standards for Mathematical Practice</b></p> <hr/> <p>As students play Total Capture with a partner, they have the opportunity to construct viable arguments and critique the reasoning of others (MP3).</p> <p>If students add in different ways, ask them to explain how they would use their way to find the total for their partner's equation. If students disagree about a total, or about whose total is smaller, encourage them to ask questions about their partner's reasoning.</p> </div> <div data-bbox="961 350 1350 846" style="border: 1px solid #ccc; padding: 10px; background-color: #e6f2ff;"> <p><b>Promoting the Standards for Mathematical Practice</b></p> <hr/> <p>When students listen to and evaluate their classmates' analysis of whether they can add the tens first in any two-digit number, they are constructing viable arguments and critiquing the reasoning of others (MP3). Students are explicitly asked to evaluate a statement as always, sometimes, or never true and to justify their answers. They can also use this as an opportunity to critique their classmates' arguments if they disagree.</p> <p>Ask the following questions to promote MP3:</p> <ul style="list-style-type: none"> <li>• When is your classmate correct?</li> <li>• Is there a general statement you can make that the class can agree to?</li> </ul> </div> <div data-bbox="1365 350 1730 803" style="border: 1px solid #ccc; padding: 10px; background-color: #e6f2ff;"> <p><b>Promoting the Standards for Mathematical Practice</b></p> <hr/> <p>When students estimate, calculate, and check their teammates' work for the product of multi-digit numbers during the Pass the Whiteboard activity, they are constructing viable arguments and critiquing the reasoning of others (MP3).</p> <p>Ask questions such as the following to promote MP3:</p> <ul style="list-style-type: none"> <li>• What parts of the work shown for the standard algorithm for multiplication do you question? Why?</li> <li>• What questions can you ask your teammate to make sure you understand their work?</li> </ul> </div> </div>
<p>C. Curricular materials support teachers in developing their own understanding of the Standards for Mathematical Practice, the role of the practice standards in lessons, and guidance for implementation.</p>	<p>✓</p>	<p>Promoting the Standards for Mathematical Practice margin notes point out places in the lesson where teachers can promote engagement and encourage student progress.</p> <p>Although most lessons offer opportunities for students to engage with more than one MP, this guidance identifies a focus MP in each lesson. The notes also provide lesson-specific information, ideas, and questions that teachers can use to deepen students' engagement with the focus MP. Often, the suggested questions for a particular MP repeat. This intentional repetition supports students' exposure to and understanding of the MPs in different contexts.</p> <p>The opening of each MP margin note supports teachers' understanding of the role of the practice standard in the designated part of the lesson by providing lesson-specific information about how the content aligns with the given standard.</p>
<p>D. Curricular materials connect content standards and practice standards in authentic ways.</p>	<p>✓</p>	<p>Lessons were written with content and practice standards in mind. At least one segment of every lesson includes the opportunity for students to engage in a MP. The connection between the practice standard and the content standard is made explicit through the margin notes, purpose statements, and facilitation guidance.</p> <p>For example, in level 3 module 4 lesson 19, students connect 3.MD.C.7.b and 3.MD.C.7.d to MP1 as they gather information and make a plan to solve a complex problem.</p>

		<div data-bbox="562 162 1123 430"> <p><b>Promoting the Standards for Mathematical Practice</b></p> <p>Students make sense of problems and persevere in solving them (MP1) as they determine what information is relevant when solving a real-world problem involving length, width, and area of rectangles.</p> <p>Ask the following questions to promote MP1:</p> <ul style="list-style-type: none"> <li>• What information or facts do you need to find out how many tiles Amy needs for her living room floor?</li> <li>• What is your plan for solving the problem?</li> </ul> </div> <div data-bbox="1123 162 1575 503"> <p>Amy wants to make changes to her living room. She draws a floor plan of her living room on grid paper.</p>  <p>Each <math>\square</math> represents 1 square foot.</p> <p>Use the Read–Draw–Write process to solve each problem.</p> <ol style="list-style-type: none"> <li>1. Amy wants to put tiles on the living room floor. Each tile has an area of 1 square foot. Amy buys 230 tiles.</li> </ol> <p>Did she buy enough tiles? How do you know?</p> </div>
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**5. Accessibility, Differentiation, and Engagement**

<p>A. Curricular materials intentionally promote student engagement, student-to-student discourse, and student ownership of learning.</p>	<p>✓</p>	<p>Student engagement and mathematical discourse is promoted throughout <i>Eureka Math</i><sup>2</sup> in various ways, including the following:</p> <p>Fluency activities such as counting, card sorts, choral response, and whiteboard activities are designed to be fun and quick-paced ways to begin each lesson and practice previously learned skills. Launch creates a need for the day’s learning through sample student work, images, videos, instructional routines, or real-world problems.</p> <p>Throughout the lesson, students have access to materials and tools that help them develop their own understanding. For example, the <i>Learn</i> book contains removables for whiteboard work or activities. <i>Learn</i> also includes lesson pages that relate to the content so that students can fully participate in their own learning. Finally, if the lesson does not already use a pictorial or physical model from the Didax manipulatives kit, teacher notes often point out where students may choose tools to support their own learning.</p> <p>Rather than being completely made up of teacher-directed work, lessons help teachers facilitate student-driven learning. Many lessons and lesson segments have meaningful tasks that integrate students’ interests followed by a sharing of student ideas and work. The lesson vignettes provide teachers with sample student work as well as probing, assessing, and advancing questions to facilitate these discussions.</p> <p>Curricular materials provide opportunities for students to collaborate in a variety of ways, including a mix of whole group interactions, dynamic pairings and groupings, and independent work. Additionally, lessons use the 10/2 principle: for every 10 minutes of teacher-directed content, students are given time for processing and student-to-student discourse. Often this involves a think–pair–share or turn and talk.</p> <p>Lessons employ instructional routines to support all students in engaging with and discussing mathematics with peers. These instructional routines foster a positive classroom community and provide a predictable environment for students to participate in mathematical discourse. The mathematical discourse and engagement with peers that the routines offer optimizes access, confidence, and belonging. Each routine presents a teachable set of steps so that students potentially have as much ownership over the routine as the teacher.</p>
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		<p>In addition to discussion-focused instructional routines, <i>Eureka Math</i><sup>2</sup> provides support for student discourse and collaboration through the Talking Tool. The Talking Tool provides grade-level appropriate sentence frames to guide students as they engage in meaningful discussion with other students. The Talking Tool provides a point of entry for students to reflect on the class discussion and respond thoughtfully by explaining their thought process, expressing agreement or disagreement, asking for more explanation, or summarizing. The Talking Tool appears on the inside cover of the student-facing <i>Learn</i> books. The <i>Teach</i> book frequently also refers to the Talking Tool, recommending it as a scaffold for multilingual learners in Language Support margin notes.</p>	 <p><b>Talking Tool</b></p> <p><b>Share Your Thinking</b>    I know ...    I did it this way because ...    The answer is ____ because ...    My drawing shows ...</p> <p><b>Agree or Disagree</b>    I agree because ...    That is true because ...    I disagree because ...    That is not true because ...    Do you agree or disagree with ____? Why?</p> <p><b>Ask for Reasoning</b>    Why did you ...?    Can you explain ...?    What can we do first?    How is ____ related to ____?</p> <p><b>Say It Again</b>    I heard you say ...    ____ said ...    Another way to say that is ...    What does that mean?</p>
<p>B. Curricular materials ensure all students can access grade-level mathematics.</p> <ul style="list-style-type: none"> <li>i. Lesson tasks provide multiple entry points into mathematics.</li> <li>ii. Curricular materials provide timely supports to assess and to address students' unfinished learning.</li> </ul>	<p>✓</p>	<p>Lesson components are designed with multiple points of entry, including low-floor, high-ceiling tasks, to promote engagement for all students.</p> <p>Fluency activities prepare students for the learning of the day and anticipate the learning of future lessons.</p> <p>Launch is often used to motivate the learning of new content. At times, Launch is used to explicitly connect prior knowledge to the day's learning. Launch is often crafted to be low-floor and high-ceiling which provides access to learners at all levels. Lessons anticipate a variety of skill levels and background knowledge by using a variety of instructional approaches to reach all learners: wordless videos that reduce language barriers, an instructional routine that encourages discourse and debate, analysis of sample work to either identify an error or make connections among models, or an interesting real-world problem or image.</p> <p><i>Eureka Math</i><sup>2</sup> <i>Equip</i><sup>TM</sup> ensures that teachers have the insight and resources to give all students access to grade-level content while bridging any gaps in prerequisite knowledge. Pre-Module Assessments are designed to identify gaps in foundational knowledge that could interfere with a student's success in the module. Each item on a Pre-Module Assessment is tied directly to one piece of essential foundational knowledge for the module. When students use the digital version of the Pre-Module Assessment, the platform automatically generates class-level and student-level reports. The class-level report includes the names of students who need each piece of supporting content.</p> <p>Item-aligned supporting activities provide just-in-time short bursts of instruction to support the foundational knowledge gap identified by a Pre-Module Assessment item. Supporting activities are intended to be sprinkled into instruction just before students need the foundational knowledge. These activities can be used during or outside the math period. Because the targeted instruction is incorporated in the lesson, students make connections between previous learning and new content. These connections enrich the depth of the learning and ensure its permanence.</p>	
<p>C. Curricular materials are designed with principles of</p>	<p>✓</p>	<p><i>Eureka Math</i><sup>2</sup> prioritizes equitable access for all students by using a teaching approach that provides every student an entry point to grade-level learning. Teacher-writers designed the curriculum around Universal Design for Learning (UDL) principles to provide multiple means of engagement, representation, and action and expression. CAST's Universal Design for Learning Guidelines (2018b) help educators design instruction that proactively reduces barriers to learning by designing for the predictable variability of all learners. Options that address learner variability</p>	

<p>Universal Design for Learning by providing multiple means of engagement, representation, and action and expression.</p>		<p>are built into the lesson design and suggested at point of use in margin notes.</p> <p>Here is an overview of each guiding principle, its instructional implications, and an example from the curricular materials.</p> <p>Provide Multiple Means of Engagement: Learners differ markedly in the ways they can be engaged or motivated to learn. Therefore, we use a variety of methods to engage students. For example, lesson activities provide choice, address student interest, and arrange for students to monitor their own learning, such as with goal setting, self-assessment, and reflection.</p> <ul style="list-style-type: none"> <li>• According to CAST, “individuals are engaged by information and activities that are relevant and valuable to their interests and goals.”<sup>1</sup> In a level 1 lesson, students collect data about their preferences and make a graph by representing each student choice with a cube on a number path. Sharing favorite activities through a class survey engages students by tapping into their interests and experiences. A margin note encourages teachers to tailor the sample questions to match student interests. Adjusting questions to make them more meaningful to students provides options for recruiting interest by personalizing and contextualizing the content to learners’ lives.</li> <li>• According to CAST, “feedback that orients students toward mastery and that emphasizes the role of effort and practice rather than ‘intelligence’ or inherent ‘ability’ is an important factor in guiding students toward successful long-term habits of mind.”<sup>2</sup> In a level 1 lesson, students solve an <i>add to with result unknown</i> problem by using the RDW process. A margin note prompts teachers to provide feedback that focuses on students’ effort and use of strategies. Offering timely, relevant, consequential, and practical feedback and asking students to explain their choices provides options for sustaining effort and persistence.</li> <li>• According to CAST, “the ability to self-regulate ... is a critical aspect of human development. While many individuals develop self-regulatory skills on their own, many others have significant difficulties in developing these skills.”<sup>3</sup> In a level 2 lesson, students share strategies and reason about ways they can combine addends to make an easier problem. Students make connections between the different solutions and their own work, and then try a strategy one of their classmates used. A margin note encourages teachers to provide feedback that promotes students’ belief in themselves. Encouraging students to consider a peer’s success and connecting this to the potential for their own success provides options for self-regulation by strengthening students’ belief in their own abilities.</li> </ul> <p>Provide Multiple Means of Representation: Learners differ in the ways they perceive and comprehend information that is presented to them. To address this variance, <i>Eureka Math</i><sup>4</sup> uses a variety of strategies, instructional tools, and methods when presenting information and content.</p> <ul style="list-style-type: none"> <li>• According to CAST, “information conveyed solely through sound is not equally accessible to all learners and is especially inaccessible for learners with hearing disabilities, for learners who need more time to process information, or for learners who have memory difficulties ... . To ensure that all learners have access to learning, options should be available for any information, including emphasis, presented aurally.”<sup>4</sup> In a level 3 lesson, students model, discuss, and compare two interpretations of division after working in pairs to equally share 10 crackers. A margin note encourages teachers to create and post a visual that helps students distinguish between the <i>number of groups</i> and the <i>number in each group</i>. Posting the visual <i>provides options for perception</i> by offering alternatives for auditory information and presenting the information in more than one mode.</li> <li>• According to CAST, “classroom materials are often dominated by information in text. But text is a weak format for presenting many concepts and for explicating most processes. Furthermore, text is a particularly weak form of presentation for learners who have text- or language-related disabilities.”<sup>5</sup> In a level 3 lesson, students use tape diagrams to make sense of and solve word problems about liquid volume. Working with a partner, students identify what each part of the tape diagram represents. Then students discuss how the tape</li> </ul>
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<sup>1</sup> CAST.org, UDL Guidelines, Engagement, Checkpoint 7.2

<sup>2</sup> CAST.org, UDL Guidelines, Engagement, Research for Checkpoint 8.4

<sup>3</sup> CAST.org, UDL Guidelines, Engagement, Guideline 9

<sup>4</sup> CAST.org, UDL Guidelines, Representation, Checkpoint 2

<sup>5</sup> CAST.org, UDL Guidelines, Representation, Checkpoint 2.5

		<p>diagram helps them find the solution path. Presenting the text-based word problem paired with the tape diagram is an example of a visual representation, provides options for language and symbols by making the word problem more comprehensible. In addition, visual representations are an example of an evidence-based practice, helpful for all students, particularly those with learning disabilities.</p> <ul style="list-style-type: none"> <li>According to CAST, “learning can be cognitively inaccessible when it requires the ability to select and prioritize among many elements or sources, and where there are no options for individuals who differ in that capability. One of the most effective ways to make information more accessible is to provide explicit cues or prompts that assist individuals in attending to those features that matter most while avoiding those that matter least.”<sup>6</sup> In a level 5 lesson, students are invited to brainstorm the decomposition methods they already know and represent them in a web graphic organizer that is displayed for the class and available for student reference. Graphic organizers are another example of the evidence-based practice of using visual representations<sup>7</sup>, as they provide options for comprehension by emphasizing key ideas and relationships.</li> </ul> <p>Provide Multiple Means of Action &amp; Expression: Learners differ in the ways they can navigate a learning environment and express what they know. <i>Eureka Math</i><sup>2</sup> offers students a variety of strategies and instructional tools that allow for multiple ways to demonstrate new understanding.</p> <ul style="list-style-type: none"> <li>According to CAST, “learners differ widely in their capacity to navigate their physical environment. To reduce barriers to learning that would be introduced by the motor demands of a task, provide alternative means for response, selection, and composition.”<sup>8</sup> In a level 3 lesson, students transition from concrete to pictorial representations of the break apart and distribute strategy with arrays by tracing and shading a large array on grid paper. Margin notes in this lesson suggest offering alternative materials to minimize the fine motor challenges presented by tracing and shading. Teachers are encouraged to offer students square-inch tiles and inch grid paper, which provide options for physical action and alternative ways students can complete their arrays.</li> <li>According to CAST, “there is a tendency in schooling to focus on traditional tools rather than contemporary ones. This tendency has several liabilities: 1) it does not prepare learners for their future; 2) it limits the range of content and teaching methods that can be implemented; 3) it restricts learners’ ability to express knowledge about content (assessment); and, most importantly, 4) it constricts the kinds of learners who can be successful. Current media tools provide a more flexible and accessible toolkit with which learners can more successfully take part in their learning and articulate what they know.”<sup>9</sup> In level 3, students are asked to draw an array to show 3 rows of 5. This lesson provides options for expression and communication by allowing students to show what they know in flexible and nontraditional ways, such as drawing or using stamps and physical manipulatives.</li> <li>According to CAST, “learning can be inaccessible when it requires planning and strategy development, and where there are no options for individuals who differ in such executive functions. Young children, older students in a new domain, or any student with one of the disabilities that compromise executive functions (e.g., ADHD, ADD, Autism Spectrum Disorders) often are weak at planning and strategy development and impulsive trial and error dominates their learning.”<sup>10</sup> In a level 5 lesson, students work with a partner to draw a model to solve a division problem involving fractions. Then the class discusses the reasonableness of their quotient. A margin note prompts teachers to provide guiding questions to support students in planning, monitoring, and evaluating their progress. The guiding questions provide options for executive functions by supporting students with planning, strategizing, and metacognitive thinking as they consider what worked well and what they may do differently next time.</li> </ul>
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<sup>6</sup> CAST.org, UDL Guidelines, Representation, Research for Checkpoint 3.2

<sup>7</sup> (IRIS Center 2021)

<sup>8</sup> CAST.org, UDL Guidelines, Action & Expression, Checkpoint 4.1)

<sup>9</sup> CAST.org, UDL Guidelines, Action & Expression, Checkpoint 5.2

<sup>10</sup> CAST.org, UDL Guidelines, Action & Expression, Research for Checkpoint 6.2

D. Curricular materials provide scaffolds and instructional supports for multilingual learners.

✓

*Eureka Math*<sup>2</sup> writers relied on language development research to outline and build in language support needs for multilingual learners to engage with the language-rich lessons. With the goal of supporting the clear, concise, and precise use of reading, writing, speaking, and listening in English, *Eureka Math*<sup>2</sup> supports multilingual learners through each lesson’s instructional design. It does this by including instructional best practices, support for mathematical discourse, as well as support for the different tiers of terminology. Additionally, Language Support margin notes provide just-in-time, targeted instructional recommendations to support multilingual learners.

Instructional Best Practices in *Eureka Math*<sup>2</sup>

Practice	<i>Eureka Math</i> <sup>2</sup>
Activate prior knowledge (mathematics content, terminology, contexts)	The daily Fluency and Launch lesson components activate prior knowledge to prepare students for new learning. Context videos demonstrate math concepts in a concrete or real-world context.
Provide multiple entry points to the mathematics	Recurring Notice and Wonder routines and frequent open-middle and open-ended tasks provide multiple points of entry for students to participate. The inclusion of fine art and Math Past history components engages students with math in the real world.
Use clear, concise student-facing language	Readability guidelines ensure that words are never an obstacle to math learning.
Provide strategic active processing time	Frequent mathematical discourse, core instructional routines, and the 10/2 principle expand opportunities for students to synthesize and process new information.
Illustrate multiple modes and formats	Varied physical and visual models, such as digital interactives, context videos, and graphic organizers, help students make connections and deepen understanding.
Provide opportunities for strategic review	Daily fluency activities, distributed practice Remember problems, Exit Tickets, and comprehensive assessments provide frequent opportunities for strategic review.

A Language Support margin note appears at the first lesson of every module to prompts teachers to consider using strategic, flexible grouping in each activity of the entire module to support multilingual learners. These grouping suggestions invite teachers to use students' funds of knowledge and home language by pairing students in different ways. Each of these different ways of pairing students has different benefits for multilingual learners.

### Mathematical Discourse

To support multilingual learners, lessons provide ample authentic and engaging opportunities for students to read, write, speak, and listen. *Eureka Math*<sup>2</sup> supports teachers to create language-rich classrooms by modeling teacher–student discourse and by providing suggestions for supported student-to-student discourse. Since curricula in general have an abundance of receptive language experiences (reading and listening), *Eureka Math*<sup>2</sup> focuses specific supports on language production (speaking and writing) in mathematics.

The core instructional routines that promote discourse are aligned with Stanford's Language Design Principles of supporting sense-making, optimizing output, cultivating conversation, and maximizing linguistic and cognitive meta-awareness.<sup>11</sup>

*Eureka Math*<sup>2</sup> periodically includes Language Support notes that suggest sentence frames and sentence starters to support multilingual learners in student-to-student discussions, such as those used in instructional routines. In addition, the Talking Tool contains general sentence frames and sentence starters. The Talking Tool is referred to in Language Support margin notes during times of student-to-student discourse.

### Terminology

*Eureka Math*<sup>2</sup> lessons give students experience with a new mathematical concept before naming it with a precise mathematical term. Students may see a mathematical concept come to life in a digital interactive, manipulate counters in groups, or use an instructional routine to engage in mathematical discourse before the teacher gives that concept a name. In addition, teachers are provided with educative guidance, either in the body of the lesson or in a Language Support margin note, to support students with pairing the written term with a visual representation.

*Eureka Math*<sup>2</sup> highlights domain-specific terms from previous lessons in the current lesson, along with instructional recommendations for supporting those terms. These instructional recommendations focus on previewing the meaning of the terms before students are expected to interact with them in the mathematics of the lesson. Additionally, domain-specific terms from previous lessons are also supported by pairing the written term with a visual representation.

For each grade level, we considered the academic verbs needed to engage with the mathematics. Each level in *Eureka Math*<sup>2</sup> offers a list of small, targeted academic verbs that appear in the lessons for students to preview before they are expected to understand and use the language. For example, before students are asked to *combine* in level 1, or *classify* in level 4, lessons preview the meaning of the academic verb, supporting the meaning of the term in a class discussion emphasizing the use of synonyms of that verb.

Multiple-meaning terms encompass homophones like *whole* and *hole*, and homographs, like *table* and *table*, and other pronunciation-based

### Language Support

Consider using strategic, flexible grouping throughout the module.

- Pair students who have different levels of mathematical proficiency.
- Pair students who have different levels of English language proficiency.
- Join pairs to form small groups of four.

As applicable, complement any of these groupings by pairing students who speak the same native language.

<sup>11</sup> Zwiers et al. 2017.

		<p>challenges, like the difference between <i>estimate</i> (as a noun, as in, What is your <i>estimate</i>?) and <i>estimate</i> (as a verb, as in, <i>Estimate</i> the sum.). The teacher–writers of <i>Eureka Math</i><sup>2</sup> examined lessons for multiple-meaning terms that could affect multilingual learners’ understanding of the mathematics, and included Language Support notes to preview the meaning of the term in the lesson. These previews include pairing the term with a visual, with real items, or with a video to highlight the different meanings of the term and emphasize the specific meaning used in the lesson.</p>
<p>E. Curricular materials provide differentiation suggestions for support and challenge.</p>	<p>✓</p>	<p><i>Eureka Math</i><sup>2</sup> embeds differentiation through the simple-to-complex sequencing of lesson and practice problems. This logical sequence gradually reduces supports and builds in complexity, allowing teachers to differentiate assignments for either individual or small-group work. For all students, the gradual reduction of supports and increase in complexity builds independent thinking and encourages productive struggle. Toward the end of the problem set, for example, problems are often open-ended, at Depth of Knowledge (DOK) levels 2 and 3, and integrate two or more standards and/or MPs. Teachers can assign problems of different complexities to students according to their needs or allow students to select problems in the 10 min (about) timeframe.</p> <p>Lessons provide differentiation suggestions at the point of instruction to support a wide variety of learners. Differentiation margin notes found in the <i>Teach</i> book offer guidance for adapting instruction so that all students can successfully access grade-level content. There are two types of Differentiation margin notes: Support and Challenge. Support boxes recommend scaffolds to support the learning of striving students, while Challenge boxes suggest ways to keep more advanced students engaged by providing opportunities for extension.</p>
<p>F. Curricular materials attend to social and emotional learning (SEL) competencies: self-awareness, self-management, social awareness, relationship skills, and responsible decision-making.</p>	<p>✓</p>	<p>High-quality rigorous mathematics instruction incorporates social-emotional competencies in student learning and engagement. <i>Eureka Math</i><sup>2</sup> intentionally promotes the five core competencies of social and emotional learning (SEL): self-awareness, self-management, social awareness, relationship skills, and responsible decision-making. Core social and emotional learning competencies can help students develop as learners, classroom community members, and mathematicians as well as become college- and career-ready.</p> <p><i>Learn</i>, the student edition, provides each student with a Thinking Tool and Talking Tool to support them as they work toward independently solving problems, metacognitively evaluating their progress and understanding, and interacting with others productively during collaborative work and class discussions.</p> <p>Each of the core instructional routines is aligned with the SELs:</p> <ul style="list-style-type: none"> <li>• Math Chat: Self-awareness, self-management, social awareness, responsible decision-making</li> <li>• Always Sometimes Never: Self-awareness, self-management, social awareness, relationship skills, responsible decision-making</li> <li>• Which One Doesn’t Belong? : Self-awareness, self-management, social awareness, responsible decision-making</li> <li>• Co-Construction: Self-awareness, self-management, social awareness, relationship skills, responsible decision-making</li> <li>• Critique a Flawed Response: Self-awareness, self-management, social awareness, relationship skills, responsible decision-making</li> <li>• Take a Stand: Self-awareness, self-management, social awareness, relationship skills, responsible decision-making</li> <li>• Five Framing Questions: Self-awareness, self-management, social awareness, responsible decision-making</li> <li>• Stronger Clearer Each Time: Self-awareness, self-management, social awareness, relationship skills, responsible decision-making</li> <li>• Numbered Heads: Self-awareness, social awareness, relationship skills, responsible decision-making</li> </ul>



		<p>The curriculum makes clear connections among SEL competencies and the skills called for by the Standards for Mathematical Practices. For example, when students make sense of problems and persevere in solving them (MP1), they must be aware of their own strengths and knowledge to look for an entry point (self-awareness), regulate their behavior to consider solution pathways before solving (self-management), appraise their work and time as they evaluate their progress (self-management), and listen to and respond to others' approaches (social awareness). See section 4 of this rubric for more information on SMPs.</p>
<p>G. Curricular materials are inclusive and reflect diverse cultures, ethnicities, and demographics.</p>	<p>✓</p>	<p>At Great Minds, we know that every child is capable of greatness, and we want all students to visualize themselves as mathematicians in their classrooms, today and throughout their future. Teacher-writers from across the country wrote <i>Eureka Math</i><sup>2</sup> to reflect the diverse experiences and backgrounds of students in today's classrooms.</p> <p><i>Eureka Math</i><sup>2</sup> is an inclusive mathematics curriculum that represents diverse doers of math. The curriculum's images, fine art, and pictures of people represent diversity, and problems and exercises relate to real-life experiences, perspectives, and contributions of people from various cultures, ethnicities, and gender identities. These representations reject the stereotypes and biases that have excluded many students from mathematical learning in favor of a more robust and inclusive perspective. Representing a diverse array of doers of mathematics in the curriculum inspires all students to think of themselves as mathematicians.</p> <p>For example, <i>Eureka Math</i><sup>2</sup> includes various mathematical activities that involve counting on hands or simulating a number line with one's fingers. In images throughout the curriculum, care was taken to include different skin tones and body types to model these gestures and movements. Additionally, students of various abilities are depicted throughout the curriculum.</p> <p>The animation videos feature unique characters that students will get to know, relate to, and ultimately see themselves in. Students observe the characters as they experience struggle, curiosity, achievement, pride, and even joy in the act of being mathematicians. When creating the characters for the videos, the <i>Eureka Math</i><sup>2</sup> writers asked, How can we help students engage in and relate to the mathematics in this lesson? Who needs to see themselves represented? Who needs to see themselves as a mathematician? The presence of realistic characters in a mathematical context helps students visualize the act of math in everyday life. Students come to more readily realize that they too are mathematicians, and that math surrounds them.</p> <p>Further, the names used in word problems and as sample students in the lesson vignettes are intentionally diverse to represent the wide variety of students who use the curriculum. The names in student-facing word problems are also designed for readability to ensure that they are not a barrier to accessing the math.</p> <p>To honor the contributions of many cultures to the development of math and to help all students see mathematicians of diverse cultures, identities, and abilities who look like them, <i>Eureka Math</i><sup>2</sup> includes an exciting new curriculum component, Math Past. Math Past explores the history of big ideas that helped shape the mathematics in each module. It frames mathematics as a human endeavor by telling the story of the discipline through artifacts, discoveries, and other contributions from cultures around the world. Math Past counters the traditional Eurocentric perspective and celebrates the many contributions of Black, Indigenous, and People of Color communities to the history of mathematics.</p> <p>Students draw on their own experiences in discussion. Each lesson invites students to participate in whole group dialogue, drawing on familiar experiences to inform their responses. Engaging word problems, accessible and engaging tasks, and videos throughout the curriculum relate mathematics to the experiences of students' own lives. Contexts are limited to those that are often relatable by most students, and new or less relatable contexts are supported through images, videos, or margin notes.</p> <div data-bbox="1606 703 1936 1109" data-label="Image"> <p>The image shows three rows of diverse cartoon characters. The top row is labeled 'Grade Band K-2' and features six characters of various ethnicities and abilities, including one in a wheelchair. The middle row is labeled 'Grade Band 3-5' and features six characters of various ethnicities and abilities. The bottom row is labeled 'Grade Band 6-8' and features six characters of various ethnicities and abilities.</p> </div>

<p>H. Student facing-materials are visually engaging, accessible, and readable.</p>	<p>✓</p>	<p>At Great Minds®, we believe that all students should see themselves as mathematicians and be able to engage with math text such as problem directions and word problems. Curricular materials were developed to be rigorous as well as readable and accessible so that all learners can focus on building math knowledge and understanding.</p> <p>Students who are learning to read (levels 1 and 2), students who are reading below level, readers with dyslexia, multilingual learners, students with processing challenges, and students overwhelmed by dense print, among others, all benefit from the increased accessibility and readability of the text. <i>Eureka Math</i><sup>2</sup> teacher–writers took the following measures to ensure the curriculum provides support for striving readers:</p> <ul style="list-style-type: none"> <li>• Use concise text with shorter sentence lengths.</li> <li>• Avoid contexts with less familiar multiple-meaning words (e.g., <i>consume, yield, log, revolutions</i>) and similar-sounding words (e.g., <i>sale, sail</i>).</li> <li>• Introduce content-critical terminology in context to help students build understanding.</li> <li>• Limit the introduction of new terminology that is not related to content. When language support for a new nonmath content term is needed, an image or video accompanies the problem.</li> <li>• Repeat use of familiar contexts and terminology so students spend less time making sense of context and more time on the math.</li> <li>• Repeat decodable names throughout the curriculum.</li> <li>• Optimize multiple layout components to optimize the student experience, including chunked blocks of text; images; increased white space; bullets, tables, and headings; and easily readable fonts and font sizes.</li> <li>• Use a readable font in print materials. The size of the text is developmentally appropriate—large and easy to track for emerging readers and then reduced slowly until level 3, where the size and text structure anticipates a more confident reader.</li> <li>• Include plenty of white space for students to work and show their thinking through numbers, pictures, or words.</li> </ul> <p>In levels 1 and 2, the curriculum team examined major reading foundational skills programs and synthesized a detailed two-year quarterly phonics progression to guide the writing of student pages. There are no decodability guidelines for levels 3–12. However, the curriculum still limits multisyllabic words, irregular spelling patterns, and difficult phonemes.</p> <ul style="list-style-type: none"> <li>• In levels K–1, each sentence in a student-facing prompt is separated by a line break from the next. This gives students the ability to consider each piece of information as an important part. Starting in level 2, students are expected to track sentences to the next line.</li> <li>• In levels 1 and 2, student-facing directions are kept simple, but they sometimes include necessary math content words or other words outside the readability guidelines. Visual supports are included when needed in levels 1 and 2. Teachers may read directions aloud to students as appropriate.</li> <li>• Students in kindergarten and level 1 are provided with writing support frames.</li> </ul>
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**6. Assessment Program**

<p>A. Curricular materials include frequent and varied and comprehensive assessments.</p>	<p>✓</p>	<p>All <i>Eureka Math</i><sup>2</sup> assessments formative because the assessments are intended to inform instruction. Assessments may also be summative to produce a grade or report that becomes part of a student, school, or district record. On its own, a single assessment does not show a complete picture of student progress. For example, a short assessment might use a single question to assess part of a standard, thus producing a limited perspective. A combination of assessments and data can be used to generate a complete picture of student learning or to produce a grade for the subject or course.</p>
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<p>i. Summative assessments, taken as a whole, include opportunities for students to demonstrate the full intent of grade-level standards.</p> <p>ii. Formative assessments support teachers in determining whether students met the objective(s) of the lessons and topics.</p> <p>iii. Assessment items include a combination of tasks that require students to demonstrate conceptual understanding, procedural skill and fluency, and application.</p> <p>iv. Assessment item types require students to produce a variety of answers and solutions (arguments, explanations, representations, etc.)</p>		<p>Each assessment is aligned with module-level Achievement Descriptors (see criteria 6.B) which encapsulate either all or part of a grade-level standard. This alignment ensures that the assessments reflect the balance of the standards as presented in the materials.</p> <p>The assessment system helps teachers understand student learning (what they know and can do) by generating data from many perspectives. The system at levels 3 through 5 includes</p> <ul style="list-style-type: none"> <li>• lesson-embedded Exit Tickets,</li> <li>• Topic Quizzes,</li> <li>• Pre-Module Assessments,</li> <li>• Module Assessments, and</li> <li>• Benchmark Assessments.</li> </ul> <p>The assessment system in levels K–2 is adjusted for grade-level appropriateness and includes an Observational Assessment Recording Sheet. The system includes</p> <ul style="list-style-type: none"> <li>• lesson-embedded Exit Tickets in levels 1 and 2,</li> <li>• Topic Tickets in levels 1 and 2 (a Topic Ticket replaces the Exit Ticket in the last lesson of a topic), and</li> <li>• Module Assessments in levels K–2.</li> </ul> <p>Exit Tickets are short, paper-based assessments that close lessons. They use at least one problem, question, or writing prompt to assess whether a student learned the basic skills and ideas needed for success in upcoming lessons. Items reflect the minimum proficiency that students must demonstrate to meet the lesson objective(s).</p> <p>Typical Topic Quizzes consist of four to six items that assess proficiency in the major concepts from the topic. Topic Quizzes include Depth of Knowledge (DOK) 1 and 2 items. Each Topic Quiz has three analogous digital versions; unused versions can be used for extra practice and/or retakes.</p> <p>Typical Module Assessments consist of six to 10 items that assess proficiency with the major concepts, skills, and applications taught in the module. The concepts on Module Assessments represent the primary content, but they may not assess all the standards taught in the module. Module Assessments include DOK 1, 2, and 3 items. Each Module Assessment in levels 3–5 has two analogous digital versions; unused versions can be used for extra practice and retakes.</p> <p>Each Benchmark Assessment covers the content of two modules and provides insight about students’ proficiency with the content over time. Three Benchmark Assessments are available for each level. Every Benchmark Assessment addresses all the Achievement Descriptors, or ADs, from the two covered modules. The second and third Benchmark Assessments include additional items that address a selected set of ADs from the previous two modules. The selected set of ADs is composed of standards that align with the major work of the level.</p> <p>Digital assessment items include a variety of item types, including selected response (e.g., multiple choice, choice matrix, drop-down list), technology-enhanced (drag-and-drop, hotspot, graphing, plotting), and constructed response items. Paper-based assessments in levels 1 and 2 also have a variety of question types including short answer, open-ended, and multiple select.</p> <p>Answer keys offer distractor rationales for multiple choice and multiple select items, giving teachers insight into possible reasons for a student to choose an incorrect response.</p> <p>Before developing curriculum assessments, assessment writers conducted thorough research into the item types and assessment architecture used in the PARCC and Smarter Balanced assessments and SATs, as well as in many state-level standardized tests. Additionally, items and assessment sequencing are aligned with the Cognitive Complexity Framework.</p>
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B. Assessment materials provide sufficient guidance for interpreting student performance and guiding instructional decisions.

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*Eureka Math*<sup>2</sup> features a new resource called Achievement Descriptors (ADs). ADs are standards-aligned descriptions that detail what students should know and be able to do based on the point of instruction. ADs are written using portions of standards to form a clear, concise description of the learning covered in each module. Each module has its own set of ADs; together, the sets of ADs describe what students should accomplish by the end of the year. ADs support teachers in interpreting student work in both assessments and through informal observations.

The Achievement Descriptors Overview in the *Teach* resource describes the ADs and the standard(s) addressed in each module. At the end of each module's *Teach* book, the Standards resources contain Achievement Descriptors graphics with Proficiency Indicators that describe the Achievement Descriptor, the standard with which it aligns, and indicators that a student is partially proficient, proficient, or highly proficient in that standard.

At the lesson level, the Lesson at a Glance on the first page of every lesson in the *Teach* book includes a Content Standards section that cites the Achievement Descriptors and standards that are addressed in that day's lesson.


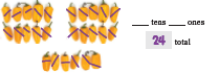


In levels 1 and 2, every module has an Observational Assessment Recording Sheet. This sheet is composed of short checklists that summarize the module's ADs. The recording sheet can be used to make notes about student performance during any part of the lesson, including written work on the Problem Set.

• **AD Code:** The code indicates the grade level and the module number and then lists the ADs in no particular order. For example, the first AD for grade 1 module 5 is coded as 1.Mod5.AD1.

• **AD Language:** The language is crafted from standards and concisely describes what will be assessed.

• **AD Indicators:** The indicators describe the precise expectations of the AD for the given proficiency category.

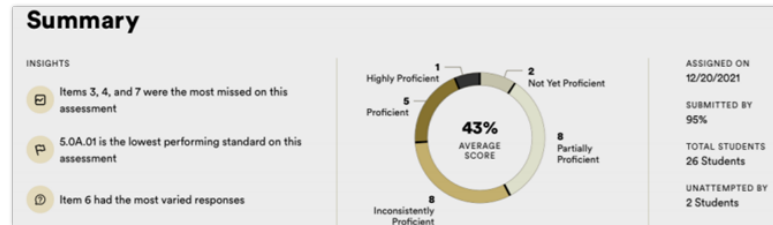
• **Related Standard:** This identifies the standards or parts of standards from the Common Core State Standards that the AD addresses.

AD Code: Grade.Module.AD#		AD Language	
1.Mod5.AD1		Represent Numbers Based on Place Value Units Use a variety of representations to model numbers in different place value situations. Compose a ten from 10 ones.	
		1.NBT.A.1, 1.NBT.B.2.a, 1.NBT.B.2.b, 1.NBT.B.2.c	
Related Standard			
Partially Proficient	Proficient	Highly Proficient	
<p>Inconsistently models to represent place value concepts. Students may</p> <ul style="list-style-type: none"> <li>show disorganized, incorrect concrete or drawn models of numbers in tens and ones, or</li> <li>count a labeled set of 10 as 1, or count all non-proportional units as 1, or</li> </ul>  <ul style="list-style-type: none"> <li>bundle fewer or more than 10, consistently count all, or</li> </ul>  <ul style="list-style-type: none"> <li>represent a number in one unit form and be unable to compose or decompose the units to model the</li> </ul>	<p>Uses a variety of models to represent an understanding of place value concepts. Composes or trades to make the next 10. Students may</p> <ul style="list-style-type: none"> <li>draw and represent numbers as amounts of tens and ones, or</li> </ul>  <ul style="list-style-type: none"> <li>accurately count objects presented in tens and ones where counting all is not an option, or</li> </ul>  <ul style="list-style-type: none"> <li>flexibly represent a number in various unit forms.</li> </ul>	<p>Deeply understands place value properties. Students may</p> <ul style="list-style-type: none"> <li>represent sets of tens or ones above 100 beyond what was taught in the lesson.</li> </ul>	
		AD Indicators	

Furthermore, *Eureka Math*<sup>2</sup> includes tools for analyzing and interpreting assessment data to measure student progress and guide lesson planning decisions. Assessment Reports interpret results, providing analysis such as the most missed items, the lowest performing standard, and the item with the most varied responses. The digital assessment reporting platform offers the following reporting features:

## Teacher Reports

- Class-Level Reports include reporting on the recent assessments completed by a particular class and assessment performance by class.
- Single Assessment Reports show which students have completed the assessment, average score, average performance by item, and a student-level breakdown of performance by item.
- Standards Mastery Reports show students' performance on all standards that have been assessed. The teacher can filter for a specific assessment or a group of assessments/all assessments for a view of overall student mastery of standards over time.
- Student Performance Reports provide details on an individual student's assessment performance compared to the class average.



## Student Reports

- Students have access to reports and can review their responses, scores, and any feedback provided by the teacher.
- Student-level reports can be downloaded and sent to parents.



## Administrator Reports

- These reports allow administrators to view school- and district-level performance data.


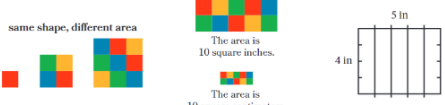
When items must be hand-scored, whether they are given digitally or in a paper-and-pencil format, guidance is provided in the assessments' Sample Solutions and Scoring Guide.

Practice problems allow students the opportunity to self-assess and provide teachers the opportunity to informally assess students' conceptual understanding, procedural skill and fluency, and application skills.

## 7. Teachability and Digital Integration

<p>A. Curricular materials include embedded and external professional development.</p> <p>i. Materials support teacher learning and understanding of mathematical concepts, the progression of learning, and instructional pedagogy.</p> <p>ii. Materials include standards alignment information and explain the role of the standards in the resources.</p>	<p>✓</p>		<p>Professional Development is available to <i>Eureka Math</i><sup>2</sup> implementers in many forms, including embedded supports in <i>TEACH</i>, digital resources, coaching and implementation services, and many professional development sessions.</p> <p>One of the great strengths of <i>Eureka Math</i><sup>2</sup> is its educative nature and its usefulness as point-of-use professional development. Extensive teacher notes throughout the curriculum provide explanations of important mathematical concepts and discussions about pedagogy, language, notation, lesson planning, and common student misconceptions.</p> <p>The <i>Teach</i> book includes module and lesson study tools for teachers. The Module Overview explains the development of the mathematics in each topic of the module and highlights connections to work that occurs before and after the module, helping teachers to understand the module’s place in the overall development of learning in and across grade levels. The Module Overview also includes a Why section. This section highlights and explains elements of the mathematics in the module. It also gives insight into the pedagogical decisions the authors made. The Why section helps teachers understand the underlying structure of the module, including the flow and coherence of the content.</p> <p>Each topic begins with a Topic Overview. The Topic Overview explains the mathematics and the major learning in the topic, and it typically includes information about how learning connects to previous or upcoming content. Lesson Progressions charts highlight student work and language aligned with each objective.</p> <p>Modeled student–teacher vignettes and sample student work in each component of each lesson provide teachers with a picture of how the lesson might look and sound while creating a clear conceptual understanding of the pedagogical content knowledge for the teacher. These vignettes are helpful guides, not scripts.</p> <p>The Standards resource lists the content standards associated with each module as well as the Standards for Mathematical Practice. Achievement Descriptors (ADs) provided for each module are standards-aligned descriptions that detail what students should know and be able to do based on instruction they receive.</p>
<p>B. Curricular materials include support for implementation.</p> <p>i. The materials are visually engaging, easy to use, and well</p>	<p>✓</p>		<p>The <i>Teach</i>, <i>Learn</i>, and <i>Apply</i> printed books help teachers facilitate student learning both inside and outside the classroom. The <i>Teach</i> book contains all the information a teacher needs to study and prepare for instruction. Similarly, the <i>Learn</i> book has all the resources a student needs to be successful when learning grade-level concepts. The curriculum also includes a student book called <i>Apply</i> that is designed for use at home. The Great Minds® Digital Platform grants teachers access to the full <i>Eureka Math</i><sup>2</sup> experience, including the contents of <i>Teach</i>, <i>Learn</i>, and <i>Apply</i>.</p> <p>This collection of resources uses consistent names and icons to assist with navigation. For example, all levels have six modules which are broken into topics and lessons. The visual design of the resources is consistent, engaging, and not distracting. The <i>Teach</i> book is spiral bound. Quality color images are used throughout in visuals, models, and sample solutions.</p> <p>The Overview in each <i>Teach</i> book is a summary of the module by topic. It shows the development of learning over the module. The Overview</p>

<p>organized for students and teachers.</p> <p>ii. Materials provide information about how to plan and prepare lessons.</p> <p>iii. Materials provide guidance for instructional delivery, including questions to prompt student thinking and expected student outcomes.</p>		<p>identifies connections to work that occurs before the module and after the module, which supports the understanding of the module’s place in the overall development of learning in and across the grade levels.</p> <p>The Why section of the Overview gives teachers insight into the decisions that authors made when writing the module. This insight helps teachers understand the underlying structure, flow of content, and coherence of content between modules and grade levels.</p> <p>The Topic Overview is a summary of the development of learning over each topic. It typically includes information about how learning connects to previous or upcoming content. It also includes a Progression of Lessons table that shows how students engage with each lesson objective over the course of the topic.</p> <p>Each lesson begins with two pages of information to help teachers prepare to teach the lesson. To guide lesson planning and instructional delivery, each lesson begins with a lesson overview that includes the Lesson at a Glance, a snapshot of the lesson framed through what students should know, understand, and do while engaging with the lesson. The lesson overview also features one or more Key Questions, which encapsulate the key learning of the lesson and develop coherence or connections to other concepts or a deeper understanding of a strategy or model. These questions help teachers focus instruction and guide classroom discourse, and students discuss them to synthesize their learning during the Land section of the lesson. In addition, the lesson overview defines clear student outcomes for each lesson by including a preview of the Exit Ticket, a formative assessment given at the end of the lesson, and the ADs, which are standards-aligned descriptions that detail what students should know and be able to do based on instruction.</p> <p>The lesson overview also details which teacher and student materials are needed for delivery of the lesson, as well as what teachers must prepare in advance of teaching the lesson. The module’s Materials resource lists items that teachers and students need for the module.</p> <p>Each lesson’s facilitation includes sample dialogue that represents how the teacher and students in a classroom might explore concepts and problems and gives a sense of how instruction might look and feel. The sample dialogue helps teachers identify lines of questioning that advance students toward the objective(s), determine when and how to use precise terminology, or navigate content that might be new or challenging to teach or learn. The sample student responses given in the lesson facilitation give teachers guidance about possible student ways of thinking and the variety of possible student responses.</p> <p>Six types of instructional guidance appear in the margin of lessons. Teacher Notes communicate information that helps with lesson implementation (e.g., enhance mathematical understanding, explain pedagogical choices, give background information, support student thinking). Margin notes also have information related to Universal Design for Learning (UDL), Language Support, Differentiation, Standards for Mathematical Practice, and Math Past.</p> <p>The <i>Eureka Math<sup>2</sup></i> Implementation Guides™, a free digital resource, are Great Minds’ comprehensive guides for teaching the curriculum. The resource provides a detailed lesson preparation method to help teachers connect all lesson components organically and to anticipate any classroom challenges. They provide a detailed overview of all curriculum components, as well as guidance for lesson preparation, sample dialogue, best practices for classroom culture, effective delivery of instructional routines, pacing guidance, and assessment overviews.</p> <p>Digital resources are intuitive and easy to use. They include a Help Center that contains articles that support teachers with navigating the digital and print resources.</p>
<p>C. Curricular materials can be completed within a regular school year and guidance about the amount of time</p>	<p>✓</p>	<p>Each grade level in <i>Eureka Math<sup>2</sup></i> is organized into six modules that contain a total of 130 to 140 lessons, along with 30 more days for assessment and responsive teaching. Optional lessons are included in the total number of lessons and are clearly designated. Lessons may be optional for the following reasons:</p> <ul style="list-style-type: none"> <li>• The lesson is primarily for enrichment.</li> <li>• The lesson offers more practice with skills, concepts, or applications.</li> <li>• The lesson bridges gaps between standards.</li> </ul>

<p>lessons and tasks may take is provided.</p>		<p>Lessons are designed for the average length of an instructional period at a given grade level. Kindergarten lessons are 50 minutes, while lessons at grade levels 1–5 are 60 minutes. All lessons are designed for one instructional period. To provide pacing guidance in the lesson, the Lesson at a Glance in the <i>Teach</i> book contains an Agenda for each lesson, detailing the amount of time each lesson component should take.</p> <p>Furthermore, the <i>Eureka Math</i><sup>2</sup> Implementation Guides contain a Year at a Glance section which outlines the modules and lessons in a year at each grade level and provides pacing suggestions for each grade-level band.</p>
<p>D. Curricular materials provide caregivers with resources to support student academic progress.</p>	<p>✓</p>	<p>Great Minds realizes that parents and family members are students’ biggest advocates and therefore works to keep them engaged in the learning process.</p> <div data-bbox="548 386 1079 678" style="border: 1px solid black; padding: 5px;"> <p>Add or subtract.</p> <p>5. <math>277 + 596 = \underline{\quad 873 \quad}</math></p>  <p>I see that the second addend, 596, is close to a benchmark number, 600.</p> <p>I can decompose the first addend, 277, into 275 and 4.</p> <p>I know <math>596 + 4 = 600</math>.</p> <math display="block">\begin{array}{r} 277 + 596 = 275 + 600 = 873 \\ \underline{275 \quad 4} \end{array}</math> <p>That makes a simpler problem, <math>275 + 600</math>.</p> </div> <p>Each <i>Apply</i> book contains Practice Partners that show problems like those worked in class and an example of the thinking that helps students solve those problems. Practice Partners serve as a useful review tool for students, as well as a guide for caregivers who are supporting their student at home.</p> <div data-bbox="1478 386 1969 834" style="border: 1px solid black; padding: 5px;"> <p>Dear Family,</p> <p>Your student is exploring area concepts. They use inch and centimeter tiles to fill in shapes without gaps or overlaps. They count each tile or skip-count to find the area. They see that polygons that are the same shape can have different areas, and they see that polygons that are different shapes can have the same area. Students learn that the size of the unit of measurement (centimeters or inches) affects the area. They transition from using tiles to representing area on grid paper. They measure the side lengths of rectangles with a ruler to discover that the number of tiles along one side of a rectangle is the same as the rectangle’s side length. Students draw a grid inside a rectangle by using their measurements and then determine the area of the rectangle.</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 30%;"> <p><b>Key Terms</b></p> <ul style="list-style-type: none"> <li>area</li> <li>side length</li> <li>square centimeter</li> <li>square inch</li> <li>square units</li> </ul> </div> <div style="width: 60%;">  <p>The amount of flat space a shape takes up is its area.</p> <p>When the side length is measured in inches, the area is measured in square inches. When the side length is measured in centimeters, the area is measured in square centimeters.</p> <p>Students use a ruler to measure the sides of a rectangle to find their lengths. Instead of using tiles to draw a rectangle on grid paper, they use a ruler to draw a grid inside the rectangle.</p> </div> </div> </div> <p>A Family Math guide for each topic provides additional overviews and ideas for home math activities. In kindergarten, the Family Math component is included in the <i>Learn</i> book.</p> <p>These resources support students in their at-home learning and are useful for anyone supporting the child’s learning including family members, tutors, and special educators.</p>
<p>E. A user-friendly online platform provides always-on access to curricular materials and additional resources.</p>	<p>✓</p>	<p>The Great Minds Digital Platform grants teachers access to the full <i>Eureka Math</i><sup>2</sup> experience. The <i>Teach</i>, <i>Learn</i>, and <i>Apply</i> print books help teachers facilitate student learning both inside and outside the classroom. A streamlined, interactive digital platform makes it simple to access and navigate <i>Eureka Math</i><sup>2</sup> teacher and student materials and resources online. The Great Minds Digital Platform supports effective planning, instruction, and assessment throughout the school year.</p> <p>Educators can view and access all levels of the <i>Eureka Math</i><sup>2</sup> curriculum via the platform. For each level, all the resources in the print <i>Teach</i> book are in a digital format that offers additional features not available in print. While planning and reflecting, educators can highlight and take notes in the digital version of <i>Teach</i> to note customizations for individuals or groups or to reflect on for future years. In addition to accessing all lesson plans and resources, teachers can assign student pages as PDFs, launch digital assessments, access assessment reports, and provide student feedback. The platform also includes lesson presentation slides that include images, student pages, videos, and digital interactives.</p> <p>Digital resources for teachers include the following:</p> <ul style="list-style-type: none"> <li>• <i>Teach</i>: Digital Teacher Edition <ul style="list-style-type: none"> <li>○ Curriculum Map</li> <li>○ Grade-level modules</li> <li>○ Module resources</li> <li>○ Topic resources</li> </ul> </li> </ul>



		<ul style="list-style-type: none"> <li>○ Lesson resources including: <ul style="list-style-type: none"> <li>● Presentation slides <ul style="list-style-type: none"> <li>● Images for lesson facilitation</li> <li>● Digital interactives to spark student curiosity, illustrate mathematical concepts, and fuel discussions</li> <li>● Context Videos</li> </ul> </li> <li>● Assign: Assign and manage any of the student facing resources in the <i>Learn</i> and <i>Apply</i> books to full classes or individual students. Monitor student progress on assignments, score and leave feedback on completed assignments, and access and edit past and upcoming assignments.</li> <li>● Assess: Use filters to search for available digital assessment in the assessment library. Assign assessments to full classes or to individual students.</li> <li>● Analyze: View assessment reports to quantitatively measure student understanding. Administrator- and teacher-facing reports provide visibility into individual and class performance on assessments and standards.</li> </ul> </li> </ul> <p>Resource Center</p> <ul style="list-style-type: none"> <li>● Announcements: Find important announcements from the <i>Eureka Math</i><sup>2</sup> and Great Minds teams. Look specifically for the What's New articles, which detail platform updates and feature releases.</li> <li>● Prototypes: Access available prototypes for user testing and feedback. The collected feedback informs further development of the product.</li> <li>● Guides: Access all walkthrough platform guidance for quick recall.</li> <li>● Help Center: View articles and videos to get answers to frequently asked platform and curriculum and implementation questions. These articles and videos are updated regularly.</li> <li>● Implementation Guides: Navigate directly to the implementation guides for the <i>Eureka Math</i><sup>2</sup> curriculum.</li> <li>● Great Minds blog: Visit the Great Minds Aha! Blog to read articles to support with implementation of <i>Eureka Math</i><sup>2</sup>.</li> </ul>
<p>F. Digital materials and experiences enhance classroom instructional practice, engaging students meaningfully to develop mathematical understanding.</p>	<p>✓</p>	<p>The student digital experience is designed to enhance the overall mathematical content learning experience. <i>Eureka Math</i><sup>2</sup> digital tools empower the teacher and invite more students into classroom discourse. Digital materials such as context videos, digital interactives, and access to online virtual manipulatives provide opportunities for students to communicate ideas and collaborate with each other.</p> <p>Lessons provide teacher guidance for the use of embedded technology to support and enhance student learning.</p> <p>Digital Resources for students include the following:</p> <ul style="list-style-type: none"> <li>● Annotation tools to complete digital assignments.</li> <li>● Tools for completing digital assessments, including various accessibility tools.</li> <li>● Access to presentation slides including context videos and digital interactives.</li> <li>● A student “locker” that contains all previously completed assignments and assessments, as well as scores and feedback left on those assignments and assessments.</li> <li>● Access to virtual manipulatives through our partner Didax.</li> </ul>

Appendix A



Year-Long Curriculum Overview: Levels K-5

STORY OF UNITS							
	Level K Part-Part-Total	Level 1 Units of Ten	Level 2 Ten Tens	Level 3 Units of Any Number	Level 4 Fractional Units	Level 5 Fractions Are Numbers	
Trimester 1	Quarter 1	Module 1: Counting and Cardinality 7 Topics   33 Lessons	Module 1: Counting, Comparison, and Addition 4 Topics   25 Lessons	Module 1: Place Value Concepts through Metric Measurement and Data • Place Value, Counting, and Comparing Within 1,000 9 Topics   38 Lessons	Module 1: Multiplication and Division with Units of 2, 3, 4, 5, and 10 5 Topics   23 Lessons	Module 1: Place Value Concepts for Addition and Subtraction 5 Topics   24 Lessons	Module 1: Place Value Concepts for Multiplication and Division with Whole Numbers 4 Topics   20 Lessons
		Quarter 2	Module 2: Two- and Three-Dimensional Shapes 3 Topics   16 Lessons		Module 2: Addition and Subtraction Relationships 5 Topics   23 Lessons	Module 2: Place Value Concepts through Metric Measurement 4 Topics   25 Lessons	Module 2: Place Value Concepts for Multiplication and Division 5 Topics   26 Lessons
Trimester 2	Quarter 3		Module 3: Comparison 4 Topics   22 Lessons	Module 3: Properties of Operations to Make Easier Problems 5 Topics   26 Lessons	Module 2: Addition and Subtraction Within 200 4 Topics   27 Lessons	Module 3: Multiplication and Division with Units of 0, 1, 6, 7, 8, and 9 4 Topics   25 Lessons	Module 3: Multiplication and Division of Multi-Digit Numbers 6 Topics   24 Lessons
		Module 4: Composition and Decomposition 3 Topics   18 Lessons	Module 4: Comparison and Composition of Length Measurements 3 Topics   14 Lessons	Module 3: Shapes and Time with Fraction Concepts 4 Topics   19 Lessons		Module 4: Multiplication and Area 4 Topics   19 Lessons	Module 4: Foundations for Fraction Operations 6 Topics   34 Lessons
Trimester 3	Quarter 4	Module 5: Addition and Subtraction 4 Topics   27 Lessons	Module 5: Place Value Concepts to Compare, Add, and Subtract 5 Topics   25 Lessons	Module 4: Addition and Subtraction Within 1,000 5 Topics   24 Lessons	Module 5: Fractions as Numbers 5 Topics   27 Lessons	Module 5: Place Value Concepts for Decimal Fractions 4 Topics   14 Lessons	Module 5: Addition and Multiplication with Area and Volume 4 Topics   28 Lessons
		Module 6: Place Value Foundations 4 Topics   24 Lessons	Module 6: Attributes of Shapes • Advancing Place Value, Addition, and Subtraction 6 Topics   31 Lessons	Module 5: Money, Data, and Customary Measurement 3 Topics   16 Lessons	Module 6: Geometry, Measurement, and Data 4 Topics   26 Lessons		Module 6: Angle Measurements and Plane Figures 4 Topics   20 Lessons
		TOTAL: 25 Topics   140 Lessons	TOTAL: 28 Topics   144 Lessons	TOTAL: 29 Topics   142 Lessons	TOTAL: 26 Topics   145 Lessons	TOTAL: 30 Topics   142 Lessons	TOTAL: 25 Topics   137 Lessons

Trimester and quarter indicators are provided as a guide for pacing. A few optional lessons in each grade level are included in the total number of lessons. About thirty additional days are allotted at each level for assessment and responsive teaching.

01/2022

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## Appendix B: Selection of *Eureka Math*<sup>2</sup> Research Citations

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