A Story of Units *TEKS Edition*
Curriculum Overview

	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	
1st TRIMESTER	M1: Numbers to 10 (43 days)	M1: Sums and Differences to 10 (45 days)	M1: Sums and Differences to 100 (10 days)	M1: Properties of Multiplication and Division and Solving Problems with Units of 2-5 and 10 (25 days)	M1: Place Value, Rounding, and Algorithms for Addition and Subtraction (25 days)	M1: Place Value and Decimal Fractions (19 days)	1st QUARTER
			M2: Addition and Subtraction of Length Units (12 days)				
	M2: Two-Dimensional and Three-Dimensional Shapes (12 days)		M3: Place Value, Counting, and Comparison of Numbers to 1,200 (24 days)	M2: Place Value and Problem Solving with Units of Measure (27 days)	M2: Unit Conversions and Problem Solving with Metric Measurement (7 days)	M2: Multi-Digit Whole Number and Decimal Fraction Operations (40 days)	
2nd TRIMESTER	M3: Comparison of Length, Weight, Capacity, and Numbers to 10 (33 days)	M2: Introduction to Place Value Through Addition and Subtraction Within 20 (35 days)	M4: Addition and Subtraction Within 200 with Word Problems to 100 (35 days)	M3: Multiplication and Division with Units of 0, 1, 6-9, and Multiples of 10 (27 days)	M3: Multi-Digit Multiplication and Division (39 days)	M3: Addition and Subtraction of Fractions (20 days)	2nd QUARTER
		M3: Ordering and Comparing Length Measurements as Numbers (15 days)	M5: Addition and Subtraction Within 1,000 with Word Problems Within 1,000 (24 days)	M4: Multiplication and Area (17 days)	M4: Angle Measure and Plane Figures (21 days)	M4: Multiplication and Division of Fractions (31 days)	
	M4: Number Pairs, Addition and Subtraction to 10 (47 days)	M4: Place Value, Comparison, Addition and Subtraction to 40 (35 days)	M6: Foundations of Multiplication, Division, and Area (24 days)	M5: Fractions as Numbers on the Number Line (35 days)	M5: Fraction Equivalence, Ordering, and Operations (37 days)	M5: Addition and Multiplication with Volume and Area (28 days)	3rd QUARTER
3rd TRIMESTER		M5: Identifying, Composing, and Partitioning Shapes (15 days)	M7: Problem Solving with Length, Money, and Data (31 days)	M6: Financial Literacy and Data (16 days)	M6: Decimal Fractions and Financial Literacy (26 days)		
	M5: Numbers 10-20, Counting to 100, and Understanding Work (34 days)	M6: Place Value, Comparison, Understanding Income with Addition and Subtraction to 100 (36 days)		M7: Geometry and Measurement Word Problems (33 days)		M6: Problem Solving with the Coordinate Plane and Data (42 days)	4th QUARTER
	M6: Analyzing, Comparing, and Composing Shapes (11 days)		M8: Time, Shapes, and Fractions as Equal Parts of Shapes (20 days)		M7: Exploring Measurement with Multiplication and Data (25 days)		

Key:			
Number	Geometry	Number and Geometry, Measurement	Fractions

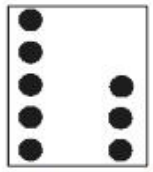
Sequence of Kindergarten Modules Aligned with the TEKS

Module 1: Numbers to 10
Module 2: Two-Dimensional and Three-Dimensional Shapes
Module 3: Comparison of Length, Weight, Capacity, and Numbers to 10
Module 4: Number Pairs, Addition and Subtraction to 10
Module 5: Numbers 10–20 and Counting to 100, and Understanding Work
Module 6: Analyzing, Comparing, and Composing Shapes

Summary of Year
Kindergarten mathematics is about (1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in Kindergarten should be devoted to numbers than to other topics.

Rationale for Module Sequence in Kindergarten
A Story of Units begins in Kindergarten. Ladybugs, fingers, and plastic bears are manipulated and counted in Kindergarten, with work consistently moving to the pictorial and abstract levels. The new, foundational unit introduced in Kindergarten's Module 5, is the supremely important unit of one. By the end of the Kindergarten year, students' first steps into place value are evidenced as they make precise statements such as, "12 is the same as 10 ones and 2 ones!" Notice how this sets the foundation for later work with decimal units (e.g., in Grade 1, "12 is the same as 1 ten and 2 ones;" in Grade 2, "12 tens is the same as 10 tens and 2 tens or 1 hundred 2 tens;" and in Grade 4, "12 tenths is the same as 10 tenths and 2 tenths or 1 one and 2 tenths").

To begin the year, Kindergarten students start out classifying and categorizing objects, leading to making one group (e.g., "I made a group of 9 goldfish. Look how I can count them in a line, in rows, and in a circle"). Students learn the way each number from 0 to 10 relates to five using fingers, cubes, drawings, 5-groups (pictured below) and the Rekenrek, an abacus with a color change after the fifth bead (pictured below). The materials support students in seeing all numbers to ten in relationship to five, as they also see them on their fingers, the best manipulative of all! This renders 6, 7, 8, 9, and 10 more friendly as they see, for example, the 3 and 5 embedded within 8. Notice how the distribution of 8 beads as 5 beads and 3 beads sets the stage for the distributive property in Grade 3 ("8 fours = 5 fours + 3 fours, so $(5 \times 4) + (3 \times 4) = 20 + 12 = 32$"). Students close the module by investigating patterns of 1 more and 1 less (excluding the word than) using models such as the number stairs (pictured below right) with a color change after the fifth cube.



5-Group Card

Rekenrek

Number Stairs

In Module 2, students take a needed break from numbers to analyze their environment and describe and identify squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. During both Modules 2 and 3, students also practice their fluency by counting and manipulating numbers to 10 during their fluency practice, giving them ample time to prepare for the addition and subtraction of Module 4.

In Module 3, students directly compare two quantities, first learning to identify the attribute being compared. The use of the word than is carefully developed first in the context of length (e.g., taller than, shorter than), then weight (heavier than, lighter than), and finally capacity. Notice how more than and less than are used to compare capacities (e.g., "The bucket holds more than the cup"). This transitions students smoothly into comparing numbers (e.g., "9 chairs is more than 6 chairs"). This concrete foundation for comparison is essential to students' entire K–12 experience. Ask any Grade 5 teacher which of the two following word problems is more challenging for students:

a) There are 34.6 kilograms of sand and 3 kilograms more gravel than sand. What is the total weight of the gravel and sand?

b) There are 34.6 kilograms of sand and 3 times as much gravel. What is the total weight of the gravel and the sand?

Problem (a) is more challenging because of the language of more than. Students consistently struggle to reason about the relationship of quantities, often resorting to using ineffective tricks (e.g., "If the problem says more than, subtract," which is not correct in the sand and gravel problem). Module 3 in Kindergarten is intended to provide a solid foundation to future comparison work in the meaningful context of measurement.

In Module 4, comparison flows into addition and subtraction, as it does in all the elementary grades (e.g., "7 is more than 3" leads to, "7 = 3 + 4," and "3 + 4 = 7"). Students represent add to, take away, and put together stories with blocks, drawings, and equations. Toward the end of the module, students start to reorient from 5 toward 10 ones with "How much more does 7 need to make ten?"

These final lessons set the stage for Module 5 wherein 10 ones is the structure on which students build the teen numbers. They are also critical foundation standards for Grade 1. Students must know how much a number needs to make ten in order to use the make ten strategy in Grades 1 and 2, shown to be an important route to place value understanding as they master their sums and differences to 20 by the end of Grade 2.

In Module 5, after an extended experience of addition and subtraction with totals up to 10, students progress to investigating numbers 10–20. For example, thirteen beans are decomposed as 10 beans and 3 beans just as 8 beans are decomposed as 5 beans and 3 beans. Students record their decompositions of the teen numbers as equations, 13 = 10 + 3, and start to think, "10. 3 more is 13." As mentioned at the beginning of the story in Grade 1, the unit one is introduced as students learn to think of the teen numbers as 10 ones and some ones. For the first time, one is not an object but rather a noun! Notice how this sets the stage for expanded form in the upper grades (e.g., 36 = 30 + 6, or 13.6 = 10 + 3 + 0.6).

Module 6 rounds out the year with an exploration of shapes. Students build shapes from components, analyze and compare them, and discover that they can be composed of smaller shapes, just as larger numbers are composed of smaller numbers.

Sequence of Grade 1 Modules Aligned with the TEKS

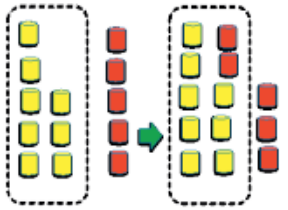
Module 1: Sums and Differences to 10
Module 2: Introduction to Place Value Through Addition and Subtraction Within 20
Module 3: Ordering and Comparing Length Measurements as Numbers
Module 4: Place Value, Comparison, Addition and Subtraction to 40
Module 5: Identifying, Composing, and Partitioning Shapes
Module 6: Place Value, Comparison, Understanding Income with Addition and Subtraction to 100

Summary of Year
Grade 1 mathematics is about (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Rationale for Module Sequence in Grade 1

In Grade 1, work with numbers to 10 continues to be a major stepping-stone in learning the place value system. In Module 1, students work to further understand the meaning of addition and subtraction begun in Kindergarten, largely within the context of the Grade 1 word problem types. They begin intentionally and energetically building fluency with addition and subtraction facts—a major gateway to later grades.

In Module 2, students add and subtract within 20. Work begins by modeling adding and subtracting across ten in word problems and with equations. Solutions involving decomposition and composition like that shown to the right for 8 + 5 reinforce the need to make 10. In Module 1, students grouped 10 objects, saw numbers 0 to 9 in relationship to ten, added to make ten, and subtracted from ten. They now transition to conceptualizing that ten as a single unit (e.g., using 10 linking cubes stuck together). This is the next major stepping-stone in understanding place value, learning to group 10 ones as a single unit: 1 ten. Learning to complete a unit empowers students in later grades to understand renaming in the addition algorithm, to add 298 and 35 mentally (i.e., 298 + 2 + 33), and to add measurements like 4 m, 80 cm, and 50 cm (i.e., 4 m + 80 cm + 20 cm + 30 cm = 4 m + 1 m + 30 cm = 5 m 30 cm).



$8 + 5 = 8 + (2 + 3) = (8 + 2) + 3 = 10 + 3 = 13$

Adding Across a Ten

Module 3, which focuses on measuring and comparing lengths indirectly and by iterating length units, gives students a few weeks to practice and internalize making a 10 during daily fluency activities.

Module 4 returns to understanding place value. Addition and subtraction within 40 rest on firmly establishing a ten as a unit that can be counted, first introduced at the close of Module 2. Students begin to see a problem like 23 + 6 as an opportunity to separate the 2 tens in 23 and concentrate on the familiar addition problem 3 + 6. Adding 8 + 5 is related to solving 28 + 5; complete a unit of ten and add 3 more.

In Module 5, students think about attributes of shapes and practice composing and decomposing geometric shapes. They also practice working with addition and subtraction within 40 during daily fluency activities (from Module 4). Thus, this module provides

important internalization time for students between two intense number-based modules. The module placement also gives more spatially-oriented students the opportunity to build their confidence before they return to arithmetic.

Although Module 6 focuses on adding and subtracting within 100, the learning goal differs from the "within 40" module. Here, the new level of complexity is to build off the place value understanding and mental math strategies that were introduced in earlier modules. Students explore by using simple examples and the familiar units of 10 made out of linking cubes, bundles, and drawings. Students also count to 120 and represent any number within that range with a numeral. Students also explore income and gifts, spending and saving, needs and wants, and charitable giving. They will continue their work of adding and subtracting within 20 in the context of financial literacy,

Sequence of Grade 2 Modules Aligned with the TEKS

Module 1: Sums and Differences to 100
Module 2: Addition and Subtraction of Length Units
Module 3: Place Value, Counting, and Comparison of Numbers to 1,200
Module 4: Addition and Subtraction Within 200 with Word Problems to 100
Module 5: Addition and Subtraction Within 1,000 with Word Problems within 1000
Module 6: Foundations of Multiplication, Division, and Area
Module 7: Problem Solving with Length, Money, and Data
Module 8: Time, Shapes, and Fractions as Equal Parts of Shapes

Summary of Year

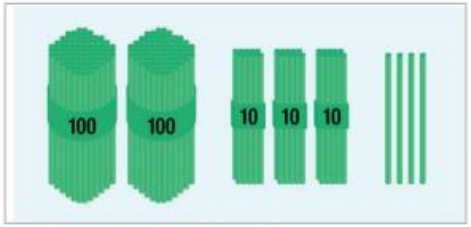
Grade 2 mathematics is about (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Rationale for Module Sequence in Grade 2

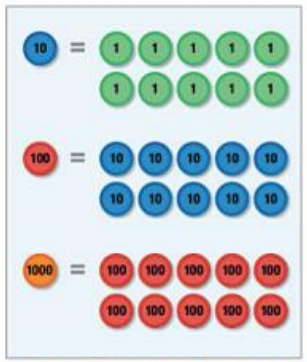
From Grade 1, students have fluency of addition and subtraction within 10 and extensive experience working with numbers to 100. Module 1 of Grade 2 establishes a motivating, differentiated fluency program in the first few weeks that will provide each student with enough practice to achieve mastery of the new required fluencies (i.e., adding and subtracting within 20 and within 100) by the end of the year. Students also solve all addition and subtraction word problem situations (See the Standards Glossary, Table 1) that do not involve comparison using the Read-Draw-Write process, a practice that will also continue throughout the year. Though encouraged to use math drawings that are intuitive for them, each situation is also modeled using the strip diagram, encouraging students to generalize and analyze part–whole relationships.

In Module 2, students learn to measure, and estimate using standard units for length and solve measurement problems involving addition and subtraction of length, now encountering the word problem situations involving comparison. A major objective is for students to use measurement tools with the understanding that linear measure involves an iteration of units and that the smaller a unit, the more iterations are necessary to cover a given length. Students work exclusively with metric units (e.g., centimeters and meters) in this module to support upcoming work with place value concepts in Module 3. Units also play a central role in the addition and subtraction algorithms of Modules 4 and 5. An underlying goal for this module is for students to learn the meaning of a unit in a different context, that of length. This understanding serves as the foundation of arithmetic, measurement, and geometry in elementary school. Students also solve word problems involving all addition and subtraction comparison situations, so that by the end of Module 2, they have encountered the full set of situations.

All arithmetic algorithms are manipulations of place value units: ones, tens, hundreds, etc. In Module 3, students extend their understanding of base ten notation and apply their understanding of place value to count and compare numbers to 1,200. In Grade 2, the place value units move from a proportional model to a non-proportional number disk model (see the pictures below). The place value table with number disks can be used through Grade 5 for modeling very large numbers and decimals, thus providing students greater facility with, and understanding of, mental math and algorithms.



Proportional Model for Place Value



Non-Proportional Model for Place Value

In Module 4, students apply their work with place value units to add and subtract within 200, moving from concrete to pictorial to abstract. This work deepens their understanding of base ten, place value, and the properties of operations. It also challenges them to apply their knowledge to one-step and two-step word problems. During this module, students also continue to develop one of the required fluencies of the grade: addition and subtraction within 100.

Module 5 builds upon the work of Module 4. Students again use place value strategies, manipulatives, and math drawings to extend their conceptual understanding of the addition and subtraction algorithms to numbers within 1,000. They maintain addition and subtraction fluency within 100 through daily application work to solve one- and two-step word problems of all types. A key component of Modules 4 and 5 is that students use place value reasoning to explain why their addition and subtraction strategies work.

In Module 6, students extend their understanding of a unit to build the foundation for multiplication and division wherein any number, not just powers of ten, can be a unit. Making equal groups of four apples each establishes the unit four apples (or just four) that can then be counted: 1 four, 2 fours, 3 fours, etc. Relating the new unit to the one used to create it lays the foundation for multiplication: 3 groups of 4 apples equal 12 apples (or 3 fours is 12). Students compose and manipulate rows and columns of an array to deepen their understanding of spatial structuring as they build and partition rectangles with rows and columns of same-size squares, thus discovering area.

Module 7 provides another opportunity for students to practice their algorithms and problem-solving skills with perhaps the most well-known, interesting units of all: dollars, dimes, pennies, quarters, and nickels. Measuring and estimating length is revisited in this module in the context of units from both the customary system (e.g., inches and feet) and the metric system (e.g., centimeters and meters). As they study money and length, students represent data given by measurement and money data using picture graphs, bar graphs, and line plots.

Students finish Grade 2 by describing and analyzing shapes in terms of their sides and angles. In Module 8, students investigate, describe, and reason about the composition and decomposition of shapes to form other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Sequence of Grade 3 Modules Aligned with the TEKS

Module 1: Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
Module 2: Place Value and Problem Solving with Units of Measure
Module 3: Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Financial Literacy and Data
Module 7: Geometry and Measurement Word Problems

Summary of Year

Grade 3 mathematics is about (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Rationale for Module Sequence in Grade 3

The first module builds upon the foundation of multiplicative thinking with units started in Grade 2. First, students concentrate on the meaning of multiplication and division and begin developing fluency for learning products involving factors of 2, 3, 4, 5, and 10 (see Key Areas of Focus and Required Fluency above). The restricted set of facts keeps learning manageable, and also provides enough examples to do one- and two-step word problems and to start measurement problems involving weight, capacity, and time in the second module.

Module 2 focuses on measurement of time and metric weight and capacity. In exploratory lessons, students decompose a kilogram into 100 gram, 10 gram, and 1 gram weights and decompose a liter into analogous amounts of milliliters. Metric measurement thereby develops the concept of mixed units (e.g., 3 kilograms 400 grams is clearly related to 3 thousands, 4 hundreds). Students then apply their new understanding of number to place value, comparison and rounding, composing larger units when adding, decomposing into smaller units when subtracting. Students also draw proportional strip diagrams to solve word problems (e.g., “If

this strip represents 62 kg, then a strip representing 35 kg needs to be slightly longer than half the 62 kg bar …"). Drawing the relative sizes of the lengths involved in the model prepares students to locate fractions on a number line in Module 5 (where they learn to locate points on the number line relative to each other and relative to the whole unit). Module 2 also provides students with internalization time for learning the 2, 3, 4, 5, and 10 facts as part of their fluency activities.

Students learn the remaining multiplication and division facts in Module 3 as they continue to develop their understanding of multiplication and division strategies within 100 and use those strategies to solve two-step word problems. The "2, 3, 4, 5, and 10 facts" module (Module 1) and the "0, 1, 6, 7, 8, 9, and multiples of 10 facts" module (Module 3) both provide important, sustained time for work in understanding the structure of rectangular arrays to prepare students for area in Module 4. This work is necessary because students initially find it difficult to distinguish the different units in a grid (the third array in the picture below), count them, and recognize that the count is related to multiplication. Tiling also supports a correct interpretation of the grid. Modules 1 and 3 slowly build up to the area model (the fourth model in the picture below), using rectangular arrays in the context of learning multiplication and division:

Progression from Rectangular Array to Area Model

By Module 4, students are ready to investigate area. They measure the area of a shape by finding the total number of same-size units of area (e.g., tiles) required to cover the shape without gaps or overlaps. When that shape is a rectangle with whole number side lengths, it is easy to partition the rectangle into squares with equal areas (as in the third stage of the illustration above).

One goal of Module 5 is for students to transition from thinking of fractions as area or parts of a figure to points on a number line and finally, as numbers. To make that jump, students think of fractions as being constructed out of unit fractions: *1 fourth* is the length of a segment on the number line such that the length of four concatenated fourth segments on the line equals 1 (the whole). Once the unit *1 fourth* has been established, counting them is as easy as counting whole numbers: 1 fourth, 2 fourths, 3 fourths, 4

fourths, 5 fourths, etc. Students also compare fractions, find equivalent fractions in special cases, and solve problems that involve fractions. They realize that equivalent fractions share the same point on the number line.
In Module 6, by applying their knowledge of fractions from Module 5, students round lengths to the nearest halves and fourths of an inch and record that information on line plots. This module also prepares students for the multiplicative comparison problems of Grade 4 by asking students "how many more" and "how many less" questions about scaled bar graphs.
The year rounds out with plenty of time to solve two-step word problems involving the four operations and to improve fluency for concepts and skills initiated earlier in the year. In Module 7, students also describe, analyze, and compare properties of two-dimensional shapes. By now, students have done enough work with both linear and area measurement models to understand that there is no relationship in general between the area of a figure and its perimeter, which is one of the concepts taught in the last module.

Module 6 builds on Grade 2 concepts about data, graphing, and dot plots. Students are given opportunities to expand upon familiar skills from Kindergarten through Grade 2 to count collections of bills and coins.

Sequence of Grade 4 Modules Aligned with the TEKS

Module 1: Place Value, Rounding, and Algorithms for Addition and Subtraction
Module 2: Unit Conversions and Problem Solving with Metric Measurement
Module 3: Multi-Digit Multiplication and Division
Module 4: Angle Measure and Plane Figures
Module 5: Fraction Equivalence, Ordering, and Operations
Module 6: Decimal Fractions and Financial Literacy
Module 7: Exploring Measurement with Multiplication and Data

Summary of Year

Grade 4 mathematics is about (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3)

understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Rationale for Module Sequence in Grade 4

In Grade 4, students extend their work with whole numbers. They begin with large numbers using familiar units (tens and hundreds) and develop their understanding of thousands by building knowledge of the pattern of *times ten* in the base-ten system on the place value chart. In Grades 2 and 3, students focused on developing the concept of composing and decomposing place value units within the addition and subtraction algorithms. Now, in Grade 4, those (de)compositions are seen through the lens of multiplicative comparison (e.g., 1 thousand is 10 times as much as 1 hundred). They next apply their broadened understanding of patterns on the place value chart to compare, round, add, and subtract. The addition and subtraction algorithms are then efficient and useful applications of students' knowledge of and skill with composing and decomposing higher value units. The module culminates with solving multi-step word problems involving addition and subtraction modeled with strip diagrams that focus on numerical relationships.

The algorithms continue to play a part in Module 2 as students relate place value units to metric units. This module helps students draw similarities between:

1 ten	= 10 ones
1 hundred	= 10 tens
1 hundred	= 100 ones
1 meter	= 100 centimeters
1 thousand	= 1,000 ones
1 kilometer	= 1,000 meters
1 kilogram	= 1,000 grams
1 liter	= 1,000 milliliters

Students work with metric measurement in the context of the addition and subtraction algorithms, mental math, place value, and word problems. Customary units are used as a context for fractions in Modules 5 and 7.

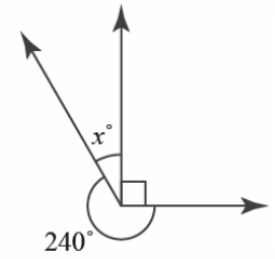
In Module 3, measurement of perimeter and area provide the concrete foundation behind the distributive property in the multiplication algorithm: 4 × (1 m 2 cm) can be modeled concretely using ribbon, since it is easy to see the 4 copies of 1 meter and the 4 copies of 2 centimeters. Likewise, 4 × (1 ten 2 ones) = 4 tens 8 ones. Students next use place value disks to develop efficient procedures and the algorithms for multiplying and dividing one-digit whole numbers. They understand and explain why the procedures work, and connections are made between the area model and work on the place value chart. Two-digit by two-digit multiplication is then modeled using the area model, extending students' earlier experiences with measurement and the distributive property. Students also solve word problems throughout the module where they select and accurately apply appropriate methods to estimate, mentally calculate, or use written strategies to compute products and quotients.

Module 4 focuses as much on solving unknown angle problems using letters and equations as it does on building, drawing, and analyzing two- dimensional shapes in geometry. Students have already used letters and equations to solve word problems in earlier grades. They continue to do so in Grade 4, and now they also learn to solve unknown angle problems: work that challenges students to build and solve equations to find unknown angle measures. First, students learn the definition of degree and learn how to measure angles in degrees using a circular protractor. From the definition of degree and the fact that angle measures are additive, the following rudimentary facts about angles naturally follow:

1. The sum of angle measurements around a point is 360 degrees.
2. The sum of angle measurements on a line is 180 degrees.

Hence, from 1 and 2, students see that vertical angles are equal. Armed only with these facts, students are able to generate and solve equations as in the following problem:

Find the unknown angle x.



$x + 240 + 90 = 360$

$x + 330 = 360$

$x = 30$

The unknown angle is 30 degrees.

Unknown angle problems help to unlock algebraic concepts for students because such problems are visual. The xx clearly stands for a specific number. If a student wished, he could place a protractor down on that angle and measure it to find xx. But doing so destroys the joy of deducing the answer and solving the puzzle on his own.

Module 5 centers on equivalent fractions and operations with fractions. We use fractions when there is a given unit, the *whole unit*, but we want to measure using a smaller unit, called the *fractional unit*. To prepare students to explore the relationship between a fractional unit and its whole unit, examples of such relationships in different contexts were already carefully established earlier in the year:

360 degrees in	1 complete turn
100 centimeters in	1 meter
1000 grams in	1 kilogram
1000 milliliters in	1 liter

The beauty of fractional units, once defined and understood, is that they behave just as all other units do:

- "4 × 3 fourths = 12 fourths" just as "4 × 3 meters = 12 meters"
- "3 fourths + 5 fourths = 8 fourths" just as "3 meters + 5 meters = 8 meters"

Students add and subtract fractions with like units using the area model and the number line. They multiply a fraction by a whole number where the interpretation is as repeated addition (e.g., 3 fourths + 3 fourths = 2 × 3 fourths). Through this introduction to fraction arithmetic they gradually come to understand fractions as units they can manipulate, just like whole numbers. Throughout the module, customary units of measurement provide a relevant context for the arithmetic.

Module 6, on decimal fractions, starts with the realization that decimal place value units are simply special fractional units: 1 tenth = 1/10, 1 hundredth = 1/100, etc. Fluency plays an important role in this topic as students learn to relate 3/10 = 0.3 = 3 tenths. They also recognize that 3 tenths is equal to 30 hundredths and subsequently have their first experience adding and subtracting fractions with unlike units (e.g., 3 tenths + 4 hundredths = 30 hundredths + 4 hundredths).

After the study of decimal notation and money, students move to the study of personal financial literacy. Students investigate different types of financial institutions, explore fixed and variable expenses, and solve word problems that calculate profits.

The year ends with a module focused on multiplication and measurement, as they solve multi-step word problems. Exploratory lessons support conceptual understanding of the relative sizes of measurement units. Students explore conversion in hands-on settings and subsequently apply those conversions to solve multi-step word problems involving all operations and multiplicative comparison.

Sequence of Grade 5 Modules Aligned with the TEKS

Module 1: Place Value and Decimal Fractions
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition and Subtraction of Fractions
Module 4: Multiplication and Division of Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Problem Solving with the Coordinate Plane

Summary of Year

Grade 5 mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Rationale for Module Sequence in Grade 5

Students' experiences with the algorithms as ways to manipulate place value units in Grades 2–4 really begin to pay dividends in Grade 5. In Module 1, whole number patterns with number disks on the place value chart are easily generalized to decimal numbers. As students work word problems with measurements in the metric system, where the same patterns occur, they begin to appreciate

the value and the meaning of decimals. Students apply their work with place value to adding, subtracting, multiplying, and dividing decimal numbers with tenths and hundredths.

Module 2 begins by using place value patterns and the distributive and associative properties to multiply multi-digit numbers by multiples of 10 and leads to fluency with multi-digit whole number multiplication. For multiplication, students must grapple with and fully understand the distributive property (one of the key reasons for teaching the multi-digit algorithm). While the multi-digit multiplication algorithm is a straightforward generalization of the one-digit multiplication algorithm, the division algorithm with two-digit divisors requires far more care to teach because students have to also learn estimation strategies, error correction strategies, and the idea of successive approximation (all of which are central concepts in math, science, and engineering).

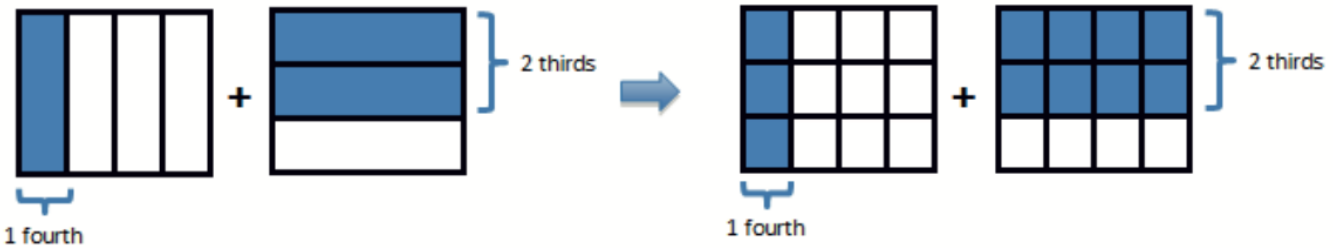
Work with place value units paves the path toward fraction arithmetic in Module 3 as elementary math's place value emphasis shifts to the larger set of fractional units for algebra. Like units are added to and subtracted from like units:

$1.5 + 0.8 = 1\frac{5}{10} + \frac{8}{10}$ = 15 tenths + 8 tenths = 23 tenths = 2 and 3 tenths = $2\frac{3}{10}$ = 2.3

$1\frac{5}{9} + \frac{8}{9}$ = 14 ninths + 8 ninths = 22 ninths = 2 and 4 ninths = $2\frac{4}{9}$

The new complexity is that when units are not equivalent, they must be changed for smaller equal units so that they can be added or subtracted. Probably the best model for showing this is the rectangular fraction model pictured below. The equivalence is then represented symbolically as students engage in active meaning-making rather than obeying the perhaps mysterious command to "multiply the top and bottom by the same number."

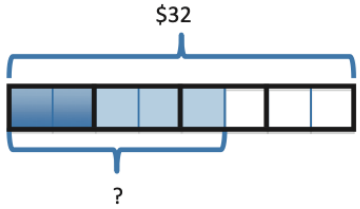
1 boy + 2 girls = 1 child + 2 children = 3 children
1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths



$$\frac{1}{4} + \frac{2}{3} = \left(\frac{1 \times 3}{4 \times 3}\right) + \left(\frac{2 \times 4}{3 \times 4}\right) = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

Relating different fractional units to one another requires extensive work with area and number line diagrams whereas strip diagrams are used often in word problems. Strip diagrams, which students began using in the early grades and which become increasingly useful as students applied them to a greater variety of word problems, hit their full strength as a model when applied to fraction word problems. At the heart of a strip diagram is the now-familiar idea of forming units. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic, as in the following example:

Jill had \$32. She gave $\frac{1}{4}$ of her money to charity and $\frac{3}{8}$ of her money to her brother. How much did she give altogether?



Solution with units:

8 units = \$32
1 unit = \$4
5 units = \$20

Solution with arithmetic:

$$\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$$

$$\frac{5}{8}\times 32 = 20$$

Jill gave \$20 altogether.

In Module 4, students learn to multiply fractions and begin the work of fraction division. Students decompose non-unit fractions and represent these decompositions as a repeated addition sentence. They will apply this understanding of decomposition and multiplication and the use of the associative property in order to multiply a whole number by a fraction. Continuing in Module 4, students learn to evaluate expressions, such as 3 times the *difference between* $\frac{2}{3}$ and $\frac{1}{5}$ *or two-thirds the sum of 7 and 9.* Towards the end of Module 4, students begin the work of division with fractions using strip diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number.

In Module 4, through real-life scenarios, students understand how to balance a simple budget.

Frequent use of the area model in Modules 3 and 4 prepares students for an in-depth discussion of area and volume in Module 5. But the module on area and volume also reinforces work done in the fraction module. Now, questions about how the area changes when a rectangle is scaled by a whole or fractional scale factor may be asked, and missing fractional sides may be found. Measuring volume once again highlights the unit theme, as a unit cube is chosen to represent a volume unit and used to measure the volume of simple shapes composed of rectangular prisms.

In this final module of *A Story of Units*, students connect plane geometry with numerical work to investigate relationships. They construct the coordinate plane, plot points and draw lines. For points on a given line, students discover a common relationship between the *x* and *y* coordinates, foreshadowing the proportional reasoning of Grade 6, and later, the slope of a line.