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GRADE 3 • MODULE 1

Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

Grade 3 • Module 1

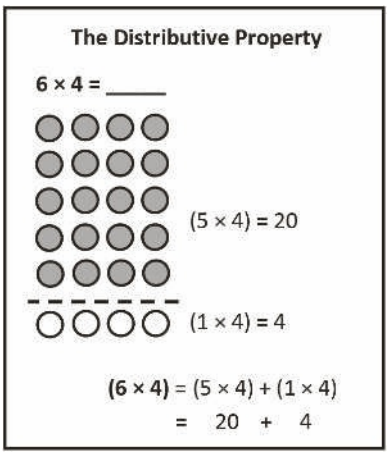
Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

OVERVIEW

This 25-day module begins the year by building on students' fluency with addition and their knowledge of arrays. In Topic A, students initially use repeated addition to find the total from a number of equal groups (**2.6A**). As students notice patterns, they let go of longer addition sentences in favor of more efficient multiplication facts (**3.4D, 3.4E, 3.4F**). Lessons in Topic A move students' Grade 2 work with arrays and repeated addition a step further by developing skip-counting rows as a strategy for multiplication. Arrays become a cornerstone of the module. Students use the language of multiplication as they understand what factors are and differentiate between the size of groups and the number of groups within a given context. In this module, the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

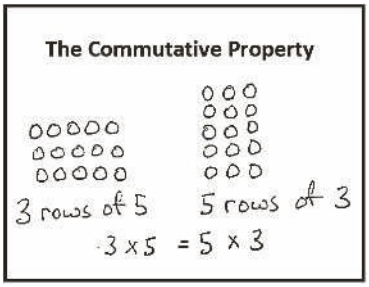
The study of factors links Topics A and B; Topic B extends the study to division. Students understand division as an unknown factor problem and relate the meaning of unknown factors to either the number or the size of groups (**3.4H, 3.4J, 3.5D**). By the end of Topic B, students are aware of a fundamental connection between multiplication and division that lays the foundation for the rest of the module.

In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication and practice related facts of 2 and 3. They become fluent enough with arithmetic patterns to *add* or *subtract* groups from known products to solve more complex multiplication problems (**3.4D, 3.4E, 3.4F**). They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor (**3.4K**). This culminates in students using arrays to model the distributive property as they decompose units to multiply (**3.4K**).



In Topic D, students model, write, and solve partitive and measurement division problems with 2 and 3 (**3.4H**). Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division. Modeling advances as students use strip diagrams to represent multiplication and division. A final lesson in this topic solidifies a growing understanding of the relationship between operations (**3.4E**).

Topic E shifts students from simple understanding to analyzing the relationship between multiplication and division. Practice of both operations is combined—this time using units of 4—and a lesson is explicitly dedicated to modeling the connection between them (**3.4J**). Skip-counting, the distributive property, arrays, number bonds, and strip diagrams are tools for both operations (**3.4D, 3.4E, 3.4H**). A final lesson invites students to explore their work with arrays and related facts through the lens of the commutative property as it relates to multiplication (**3.4D**).



Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have used to mixed practice with all of the factors included in Module 1 (**3.4K**). Students model relationships between factors, analyzing the arithmetic patterns that emerge to compose and decompose numbers, as they further explore the relationship between multiplication and division (**3.4J**).

In the final lesson of the module, students apply the tools, representations, and concepts they have learned to problem solving with multi-step word problems using all four operations (**3.4K, 3.5A, 3.5B**). They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

The Mid-Module Assessment follows Topic C. The End-of-Module Assessment follows Topic F.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Consolidate Lessons 12 and 13, both of which are division lessons sharing the same objective. Include units of 2 and units of 3 in the consolidated lesson.

Omit Lessons 15 and 19. Lesson 15 uses the strip diagram to provide a new perspective on the commutative property, a concept students have studied since Lesson 7. Lesson 19 introduces the significant complexity of the distributive property with division. The concepts from both lessons are reinforced within Module 3.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy.[1] The student is expected to:

3.4D determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by;

3.4E represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip-counting;

3.4F recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;

3.4H determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;

3.4J determine a quotient using the relationship between multiplication and division;

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations[2]; or recall of facts.

Algebraic Reasoning

The student applies mathematical process standards to analyze and create patterns and relationships.[3] The student is expected to:

3.5A represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;

3.5B represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;

3.5D determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product.

[1]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

[2]The associative property is addressed in Module 3.

[3]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

Foundational Standards

The student is expected to:

2.2C generate a number that is greater than or less than a given whole number up to 1,200;

2.6A model, create and describe contextual multiplication situations in which equivalent sets of concrete objects are joined;

2.7A determine whether a number up to 40 is even or odd using pairings of objects to represent the number;

2.9F use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(A) apply mathematics to problems arising in everyday life, society, and the workplace;

MPS(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS		Topics and Objectives	Days
3.4D **3.4E** **3.4F** 3.4K	1.C 1.E 2.E 2.I 3.E 4.D 4.G 5.B	A	**Multiplication and the Meaning of the Factors** Lesson 1: Understand *equal groups of* as multiplication. Lesson 2: Relate multiplication to the array model. Lesson 3: Interpret the meaning of factors—the size of the group or the number of groups.	3
3.4H **3.4J** **3.5D** 3.4K	1.A 1.C 2.E 2.I 3.F 4.D 4.G 5.B	B	**Division as an Unknown Factor Problem** Lesson 4: Understand the meaning of the unknown as the size of the group in division. Lesson 5: Understand the meaning of the unknown as the number of groups in division. Lesson 6: Interpret the unknown in division using the array model.	3
3.4D **3.4E** **3.4F** **3.4K** 3.5D	1.C 1.F 2.C 2.I 3.B 3.E 3.H 4.G 5.B	C	**Multiplication Using Units of 2 and 3** Lessons 7–8: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models. Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models. Lesson 10: Model the distributive property with arrays to decompose units as a strategy to multiply.	4
			Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
3.4E **3.4F** **3.4H** **3.4J** **3.5D** 3.4K 3.5A 3.5B	1.C 1.E 2.G 2.I 3.C 4.D 4.G 5.A 5.B 5.C	D	**Division Using Units of 2 and 3** Lesson 11: Model division as the unknown factor in multiplication using arrays and strip diagrams. Lesson 12: Interpret the quotient as the number of groups or the number of objects in each group using units of 2. Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.	3

TEKS	ELPS	Topics and Objectives		Days
3.4E **3.4K** 3.4D 3.4H 3.4J 3.5D	1.C 1.F 1.G 2.E 2.I 3.D 3.E 3.H 4.G 5.B	E	**Multiplication and Division Using Units of 4** Lesson 14: Skip-count objects in models to build fluency with multiplication facts using units of 4. Lesson 15: Relate arrays to strip diagrams to model the commutative property of multiplication. Lesson 16: Use the distributive property as a strategy to find related multiplication facts. Lesson 17: Model the relationship between multiplication and division.	4
3.4E **3.4F** **3.4K** **3.5A** **3.5B** 3.4D 3.4H 3.4J 3.5D	1.C 2.A 2.G 2.I 3.E 3.G 3.H 4.D 4.G 5.B 5.E	F	**Distributive Property and Problem Solving Using Units of 2–5 and 10** Lessons 18–19: Apply the distributive property to decompose units. Lesson 20: Solve two-step word problems involving multiplication and division, and assess the reasonableness of answers. Lesson 21: Solve two-step word problems involving all four operations, and assess the reasonableness of answers.	4
			End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days				**25**

Terminology

New or Recently Introduced Terms

- Array[4] (arrangement of objects in rows and columns)
- Commutative property/commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Distribute (with reference to the distributive property, e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$ the 3 is the multiplier for each part of the decomposition)
- Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)

[4]Originally introduced in Grade 2, Module 6 but treated as new vocabulary in this module.

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GRADE 3 • MODULE 2

Place Value and Problem Solving with Units of Measure

Grade 3 • Module 2

Place Value and Problem Solving with Units of Measure

OVERVIEW

In this 27-day module, students explore measurement using kilograms, grams, liters, milliliters, and intervals of time in minutes. They understand time as a continuous measurement through exploration with stopwatches, and use the number line, a continuous measurement model, as a tool for counting intervals of minutes within 1 hour (**3.7C**). Students see that an analog clock is a portion of the number line shaped into a circle. They use both the number line and clock to represent addition and subtraction problems involving intervals of minutes within 1 hour (**3.7C**).

Introduced in Topic B, kilograms and grams are measured using digital and spring scales. Students use manipulatives to build a kilogram and then decompose it to explore the relationship between the size and weight of kilograms and grams (**3.7D, 3.7E**). An exploratory lesson relates metric weight and liquid volume measured in liters and milliliters, highlighting the coherence of metric measurement. Students practice measuring liquid volume using the vertical number line and a graduated beaker (**3.7D, 3.7E**). Building on the estimation skills with metric length gained in Grade 2, students in Grade 3 use kilograms, grams, liters, and milliliters to estimate the weights and liquid volumes of familiar objects. Finally, they use their estimates to reason about solutions to one-step addition, subtraction, multiplication, and division word problems involving metric weight and liquid volume given in the same units (**3.7D, 3.7E**).

In Topic C, students extend their understanding of place value from Grade 2 (**2.2B–E**) to name numbers up to 100,000 (**3.2A**). The place value chart is fundamental to Topic C. Building upon their previous knowledge, students learn that 10 hundreds can be composed into 1 thousand, and 10 thousands can be composed into 1 ten thousand. Students represent these numbers in various forms including base ten numerals, number names, expanded form, and expanded notation (**3.2A, 3.2B**). Student then use place value as a basis for comparing whole numbers using the symbols <, >, and = to record the comparison (**3.2D**).

Now more experienced with measurement and estimation using different units and tools, students further develop their skills by learning to round in Topic D. They measure and then use place value understandings and the number line as tools to round up to five-digit measurements (**3.4B**).

Students measure and round to solve problems in Topics E and F. In these topics, they use estimations to test the reasonableness of sums and differences precisely calculated using standard algorithms. From their work with metric measurement,[1] students have a deeper understanding of the composition and decomposition of units. They demonstrate this understanding in every step of the addition and subtraction algorithms with two- and three-digit numbers, as 10 units are changed for 1 larger unit or 1 larger unit is changed for 10 smaller units (**3.2A**). Both topics end in problem solving involving metric units or intervals of time. Students round to estimate and then calculate precisely using the standard algorithm to add or subtract two- and three-digit measurements given in the same units (**3.4A, 3.4B**).

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Consolidate Lessons 17 and 18. Within the lesson that results, include some problems that require regrouping once to add and some problems that require regrouping twice.

Consolidate Lessons 20 and 21. Within the lesson that results, include some problems that require regrouping once to subtract and some problems that require regrouping twice.

Omit Lesson 22. While it engages students in a study of estimation and provides practice with reasoning about the relationships between quantities, the lesson does not present new skills.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:

3.2A compose and decompose numbers up to 100,000 as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate;

3.2B describe the mathematical relationships found in the base-10 place value system through the hundred thousands place;

3.2C represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words to describe relative size of numbers in order to round whole numbers;

3.2D compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.

[1]Students work with customary units in Modules 4 and 6.

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

3.4A solve with fluency one-step and two-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction;

3.4B round to the nearest 10 or 100 or use compatible numbers to estimate solutions to addition and subtraction problems.

Geometry and Measurement

The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:

3.7C determine the solutions to problems involving addition and subtraction of time intervals in minutes using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes;

3.7D determine when it is appropriate to use measurements of liquid volume (capacity) or weight;

3.7E determine liquid volume (capacity) or weight using appropriate units and tools.

Foundational Standards

The student is expected to:

2.9A find the length of objects using concrete models for standard units of length;

2.9D determine the length of an object to the nearest marked unit using rulers, yardsticks, meter sticks, or measuring tapes;

2.9G read and write time to the nearest one-minute increment using analog and digital clocks and distinguish between a.m. and p.m.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS		Topics and Objectives	Days
3.4A **3.7C**	1.A 1.C 1.E 2.I 3.E 4.B 4.D 4.G	A	**Time Measurement and Problem Solving** Lesson 1: Explore time as a continuous measurement using a stopwatch. Lesson 2: Solve word problems involving time intervals within 1 hour by counting backward and forward using the number line and clock. Lesson 3: Solve word problems involving time intervals within 1 hour by adding and subtracting on the number line.	3
3.2A **3.4A** **3.7D** **3.7E**	1.A 1.C 2.C 2.I 3.F 3.H 4.D 4.G 4.H 5.B	B	**Measuring Weight and Liquid Volume in Metric Units** Lesson 4: Build and decompose a kilogram to reason about the size and weight of 1 kilogram, 100 grams, 10 grams, and 1 gram. Lesson 5: Develop estimation strategies by reasoning about the weight in kilograms of a series of familiar objects to establish mental benchmark measures. Lesson 6: Solve one-step word problems involving metric weights within 100 and estimate to reason about solutions. Lesson 7: Decompose a liter to reason about the size of 1 liter, 100 milliliters, 10 milliliters, and 1 milliliter. Lesson 8: Estimate and measure liquid volume in liters and milliliters using the vertical number line. Lesson 9: Solve mixed word problems involving all four operations with grams, kilograms, liters, and milliliters given in the same units.	6
3.2A **3.2B** **3.2D**	1.A 1.C 2.C 2.I 3.E 4.B 4.G 5.B	C	**Place Value and Comparing Multi-Digit Whole Numbers** Lesson 10: Name numbers up to 100,000 by building understanding of the place value chart and placement of commas for naming base thousand units. Lesson 11: Read and write numbers to 100,000 using base ten numerals, number names, expanded form, and expanded notation. Lesson 12: Compare numbers based on the meaning of the digits using <, >, or = to record the comparison.	3
			Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day)	2

TEKS	ELPS		Topics and Objectives	Days
3.2A **3.2B** **3.2C** **3.4B**	1.C 2.C 2.I 3.D 3.E 4.B 4.D 4.G 5.B	D	**Rounding to the Nearest Ten, Hundred, Thousand, and Ten Thousand** Lesson 13: Round two-digit measurements to the nearest ten on the vertical number line. Lesson 14: Round two- and three-digit numbers to the nearest ten on the vertical number line. Lesson 15: Round to the nearest hundred on the vertical number line. Lesson 16: Round four- and five-digit numbers using the vertical number line.	4
3.2A **3.4A** **3.4B** 3.2B 3.2C 3.7C 3.7D 3.7E	1.F 2.G 2.I 3.E 4.B 4.D 4.G 5.B	E	**Two- and Three-Digit Measurement Addition Using the Standard Algorithm** Lesson 17: Add measurements using the standard algorithm to compose larger units once. Lesson 18: Add measurements using the standard algorithm to compose larger units twice. Lesson 19: Estimate sums by rounding and apply to solve measurement word problems.	3
3.2A **3.4A** **3.4B** 3.2B 3.2C 3.7C 3.7D 3.7E	1.D 2.E 2.I 3.C 3.E 3.F 4.B 4.D 4.G 5.E	F	**Two- and Three-Digit Measurement Subtraction Using the Standard Algorithm** Lesson 20: Decompose once to subtract measurements including three-digit minuends with zeros in the tens or ones place. Lesson 21: Decompose twice to subtract measurements including three-digit minuends with zeros in the tens and ones places. Lesson 22: Estimate differences by rounding and apply to solve measurement word problems. Lesson 23: Estimate sums and differences of measurements by rounding, and then solve mixed word problems.	4
			End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
Total Number of Instructional Days				**27**

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GRADE 3 • MODULE 3

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

Grade 3 • Module 3

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

OVERVIEW

This 27-day module builds directly on students' work with multiplication and division in Module 1. At this point, Module 1 instruction coupled with fluency practice in Module 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100 (**3.4F**). Module 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Additionally students will multiply two-digit numbers by one-digit numbers, and record their work in both the partial product algorithm and the standard algorithm. Similar to the organization of Module 1, the introduction of new factors in Module 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by building on students' understanding of multiplication by exploring multiplication as a means to compare one quantity to another (**3.5C**). Students will work from simple to complex, starting with strips of paper, transitioning to strip diagrams and then using number pairs in a table. While they do this work, students are enhancing their fluency of basic multiplication and division facts (**3.4F**). The topic ends with a focus on multiplicative comparison word problems.

Topic B begins by revisiting the commutative property. Students study familiar facts from Module 1 to identify known facts using units of 6, 7, 8, and 9 (**3.4E, 3.4K**). They realize that they already know more than half of their facts by recognizing, for example, that if they know 2 × 8, they also know 8 × 2 through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module (**3.4I**). The subsequent lesson carries this study a step further; students apply the commutative property to relate 5 × 8 and 8 × 5 and then add one more group of 8 to solve 6 × 8 and, by extension, 8 × 6. The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a blank box to represent the unknown in various positions (**3.4K, 3.5D**).

Topic C introduces units of 6 and 7, factors that are well suited to skip-counting strategies and to the distributive property strategy, already familiar from Module 1. Students learn to compose up to and then over the next ten. For example, to solve a fact using units of 7, they might count 7, 14, and then mentally add 14 + 6 + 1 to make 21. This strategy previews the associative property using addition and illuminates patterns as students apply count-bys to solve problems. Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a blank box to represent the unknown in various positions within multiplication and division problems (**3.4E, 3.4K, 3.5D**).

Topic D anticipates the formal introduction of the associative property with a lesson focused on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation (**3.4K, 3.5A, 3.5B**). With this student knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 (**3.4K**). Units of 6 and 8 are particularly useful for presenting this strategy. Rewriting 6 as 2 × 3 or 8 as 2 × 4 makes shifts in grouping readily apparent (see example on next page) and also utilizes the familiar factors 2, 3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8 × 5:

$$8 \times 5 = (4 \times 2) \times 5$$

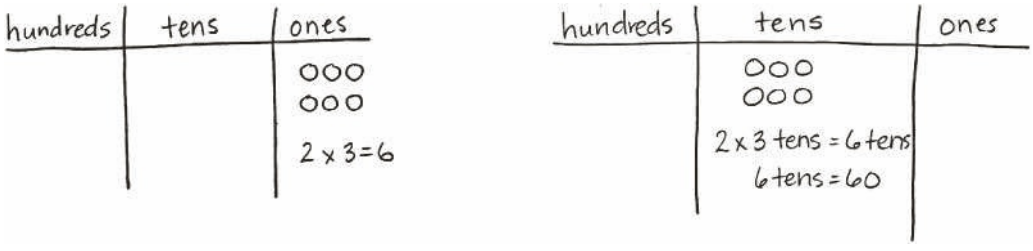
$$8 \times 5 = 4 \times (2 \times 5)$$

$$8 \times 5 = 4 \times 10$$

Topic E introduces units of 9, with students exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity (**3.4F**). Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the commutative and distributive properties to solve expressions including 9 as a factor. The topic ends with interpreting the unknown factor to solve multiplication and division problems (**3.4E, 3.4K, 3.5D**).

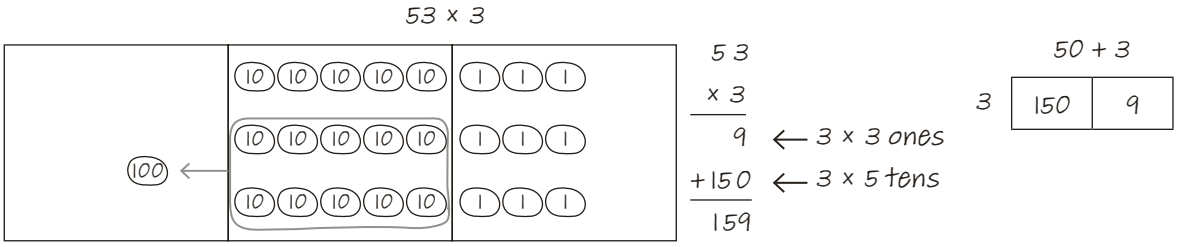
In Topic F, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students study the results of multiplying and dividing with units of 0 and 1 to identify relationships and patterns (**3.4E, 3.4I, 3.5E**). The topic closes with a lesson devoted to two-step problems involving all four operations (**3.4K, 3.5A, 3.5B**). In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions (**3.4K, 3.5A, 3.5B**).

In the first lesson of Topic G, students multiply by multiples of 10 (**3.4F, 3.4G**). To solve a problem like 2 × 30, they first model the basic fact 2 × 3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2 × 3 tens can be found by simply locating the same basic fact in the tens column.



In the subsequent lessons, students build on this understanding to multiply a two-digit number by a one-digit number (**3.4G**). Lessons proceed from concrete to abstract. In Lesson 20, students use place value disks and record work using a partial products algorithm. In Lesson 21, students continue their work of multiplying a two-digit number by a one-digit number by representing the disks on a labeled place value chart, and still

recording their work with a partial products algorithm. The standard algorithm is introduced in Lesson 22, and practiced with two-step word problems in Lesson 23 (**3.4A**). Embedded in these lessons, students continue to practice basic facts in order to gain automaticity. For example, when multiplying 6 times 4 tens, students are practicing the basic fact 6 × 4 (**3.4F**).



Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

3.4E represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;

3.4F recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division fact;

3.4G use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

3.4I determine if a number is even or odd using divisibility rules;

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

Algebraic Reasoning

The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

- **3.5A** represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;
- **3.5B** represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;
- **3.5C** describe a multiplication expression as a comparison such as 3 × 24 represents 3 times as much as 24;
- **3.5D** determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product;
- **3.5E** represent real-world relationships using number pairs in a table and verbal descriptions.

Foundational Standards

The student is expected to:

- **2.2C** generate a number that is greater than or less than a given whole number up to 1,200;
- **2.6A** model, create, describe contextual multiplication situations in which equivalent sets of concrete objects are joined;
- **2.7A** determine whether a number up to 40 is even or odd using pairings of objects to represent the number;
- **3.4D** determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10;
- **3.4E** represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;
- **3.4H** determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;
- **3.4J** determine a quotient using the relationship between multiplication and division;
- **3.5D** determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(E) a create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
3.5C **3.5E** 3.4F	1.B 1.C 1.E 2.G 3.F 4.G 5.G	A	**Multiplication as Comparison** Lessons 1–2: Use multiplication to compare. Lesson 3: Use tables to record multiplicative relationships. Lesson 4: Solve multiplicative comparison word problems.	4
3.4E **3.4K** **3.5D** **3.5E** 3.4D 3.4H	1.C 2.E 2.I 3.D 3.H 4.B 5.G	B	**The Properties of Multiplication and Division** Lesson 5: Study commutativity to find known facts of 6, 7, 8, and 9. Lesson 6: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit. Lesson 7: Multiply and divide with familiar facts using a box to represent the unknown.	3
3.4E **3.4K** **3.5D** 3.4D 3.4H	1.F 2.C 2.D 2.E 2.I 3.E 3.F 4.B	C	**Multiplication and Division Using Units of 6 and 7** Lesson 8: Count by units of 6 to multiply and divide using number bonds to decompose. Lesson 9: Count by units of 7 to multiply and divide using number bonds to decompose. Lesson 10: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.	3

TEKS	ELPS		Topics and Objectives	Days
3.4E **3.4K** **3.5D** 3.4D 3.4H 3.5A 3.5B	2.H 2.I 3.E 3.H 4.G 5.G	D	**Multiplication and Division Using Units up to 8** Lesson 11: Understand the function of parentheses and apply to solving problems. Lesson 12: Model the associative property as a strategy to multiply.	2
			Mid-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
3.4E **3.4K** **3.5D** **3.5E** 3.4D 3.4H	2.E 2.I 3.E 3.H 4.B 4.G 5.G	E	**Multiplication and Division Using Units of 9** Lesson 13: Apply the distributive property and the fact 9 = 10 – 1 as a strategy to multiply. Lesson 14: Interpret the unknown in multiplication and division to model and solve problems.	2
3.4E **3.4I** **3.4K** **3.5A** **3.5B** **3.5E** 3.4D 3.4H 3.5D	1.C 2.E 2.I 3.E 3.G 3.H 4.F 4.G 5.G	F	**Analysis of Patterns and Problem Solving Including Units of 0 and 1** Lesson 15: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division. Lesson 16: Identify patterns in multiplication and division facts using the multiplication table. Lesson 17: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.	3

TEKS	ELPS	Topics and Objectives		Days
3.4F **3.4G** **3.4K** **3.5A** **3.5B** **3.5E** 3.4D 3.4E	1.C 2.E 2.G 2.I 3.G 3.H 4.A 4.C 4.G 5.G	G	**Multiplication of Single-Digit Factors and Two-Digit Factors** Lesson 18: Multiply by multiples of 10 using the place value chart. Lesson 19: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10. Lesson 20: Use concrete models to represent two-digit by one-digit multiplication. Lesson 21: Draw models to represent two-digit by one-digit multiplication. Lesson 22: Multiply two-digit numbers by one-digit numbers using the standard algorithm. Lesson 23: Solve two-step word problems involving multiplying single-digit factors by multiples of 10 and two-digit factors.	6
			End-of-Module Assessment: Topics A–G (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days				**27**

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GRADE 3 • MODULE 4

Multiplication and Area

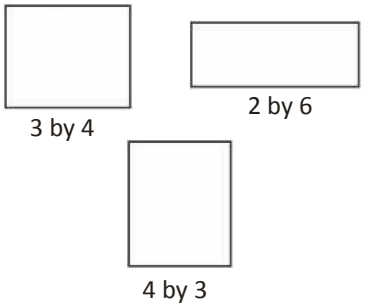
Grade 3 • Module 4

Multiplication and Area

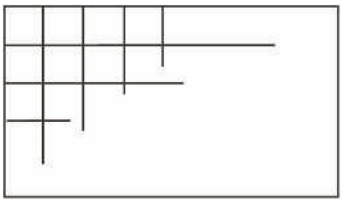
OVERVIEW

In this 17-day module, students explore area as an attribute of two-dimensional figures and relate it to their prior understandings of multiplication. In Grade 2, students partitioned a rectangle into rows and columns of same-sized squares and found the total number by both counting and adding equal addends represented by the rows or columns (**2.6A, 2.9F**).

Topic A provides students' first experience with tiling from which they learn to distinguish between length and area by placing a ruler with the same size units (inches or centimeters) next to a tiled array. They discover that the number of tiles along a side corresponds to the length of the side.

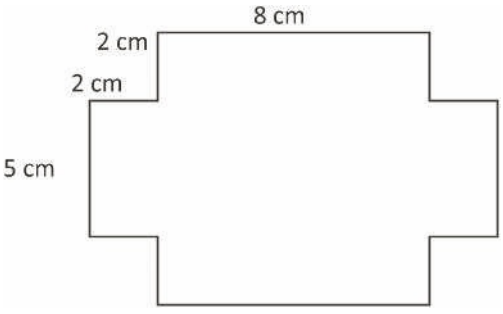


Students progress from using square tile manipulatives to drawing their own area models. Anticipating the final structure of an array, they complete rows and columns in figures such as the example shown to the right. Students connect their extensive work with rectangular arrays and multiplication to eventually discover the area formula for a rectangle, which is formally introduced in Grade 4 (**3.6C**).



In Topic B, students manipulate rectangular arrays to concretely demonstrate the arithmetic properties in anticipation of the lessons that follow. They do this by cutting rectangular grids and rearranging the parts into new wholes using the properties to validate that area stays the same, despite the new dimensions. They apply tiling and multiplication skills to determine all whole number possibilities for the side lengths of rectangles given their areas (**3.6C**).

Topic C creates an opportunity for students to solve problems involving area (**3.6C**). Students decompose or compose composite regions, such as the one shown to the right—into non-overlapping rectangles, find the area of each region, and then add or subtract to determine the total area of the original shape. This leads students to find the areas of rooms in a given floor plan (**3.6D**).



Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Consider omitting Lesson 6, which reviews previously learned skills. If omitting, be sure that students are ready to transition toward more complex practice.

Omit Lessons 12 and 13. These lessons guide students through a project involving floor plans. Skip the application of area that these lessons provide.

Focus Grade Level Standards

Geometry and Measurement

The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:

3.6C determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;

3.6D decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area.

Foundational Standards

The student is expected to:

2.6A model, create, describe contextual multiplication situations in which equivalent sets of concrete objects are joined;

2.9A find the length of objects using concrete models for standard units of length;

2.9B describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object;

2.9D determine the length of an object to the nearest marked unit using rulers, yardsticks, meter sticks, or measuring tapes;

2.9F use concrete models of square units to find the area of the rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(E) a create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

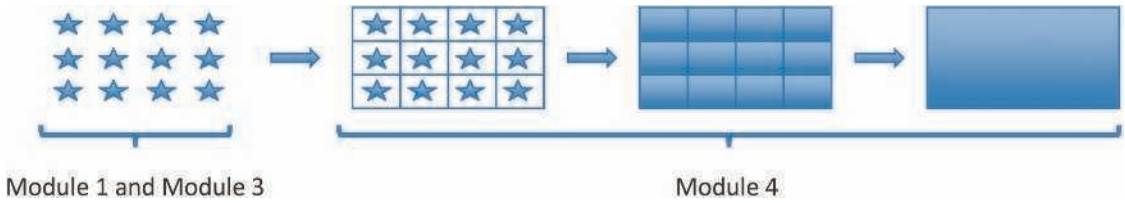
TEKS	ELPS		Topics and Objectives	Days
3.6C **3.6D** 3.4F	1.C 2.C 2.E 2.G 2.I 3.D 3.E 3.I 4.B 5.G	A	**Concepts of Area Measurement** Lesson 1: Relate side lengths to the number of tiles on a side. Lesson 2: Form rectangles by tiling with unit squares to make arrays. Lesson 3: Draw rows and columns to determine the area of a rectangle given an incomplete array. Lesson 4: Interpret area models to form rectangular arrays. Lesson 5: Find the area of a rectangle through multiplication of the side lengths.	5
			Mid-Module Assessment: Topic A (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
3.6C **3.6D** 3.4F	1.A 1.F 2.E 3.F 4.F 4.G 5.G	B	**Arithmetic Properties Using Area Models** Lesson 6: Analyze different rectangles and reason about their area. Lesson 7: Apply the distributive property as a strategy to find the total area of a larger rectangle by adding two products. Lesson 8: Demonstrate the possible whole number side lengths of rectangles with areas of 24, 36, 48, or 72 square units using the associative property.	3

TEKS	ELPS		Topics and Objectives	Days
3.6C **3.6D** 3.4F	1.A 2.E 2.I 3.E 4.G 5.G	C	**Applications of Area Using Side Lengths of Figures** Lesson 9: Solve word problems involving area. Lessons 10–11: Find areas by decomposing into rectangles or completing composite figures to form rectangles. Lessons 12–13: Apply knowledge of area to determine areas of rooms in a given floor plan.	5
			End-of-Module Assessment: Topics A–C (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
Total Number of Instructional Days				**17**

Terminology

Familiar Terms and Symbols[1]

- Area (the amount of two-dimensional space in a bounded region)
- Area model (a model for multiplication that relates rectangular arrays to area)



- Array (a set of numbers or objects that follow a specific pattern: a matrix)
- Commutative property (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Distribute (e.g., $2 \times (3 + 4) = 2 \times 3 + 2 \times 4$)
- Geometric shape (a two-dimensional object with a specific outline or form)
- Length (the straight-line distance between two points)
- Multiplication (e.g., $5 \times 3 = 15$)
- Rows and columns (e.g., in reference to rectangular arrays)
- Square unit (a unit of area—specifically square centimeters, inches, feet, and meters)
- Tile (to cover a region without gaps or overlaps)
- Unit square (e.g., given a length unit, it is a 1 unit by 1 unit square)
- Whole number (an integer, i.e., a number without fractions)

[1]These are terms and symbols students have seen previously.

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GRADE 3 • MODULE 5

Fractions as Numbers on the Number Line

Grade 3 • Module 5

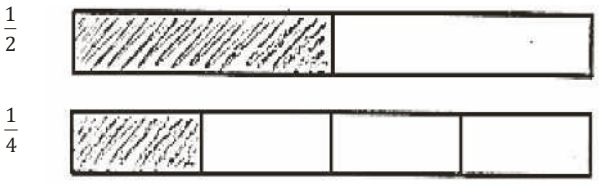
Fractions as Numbers on the Number Line

OVERVIEW

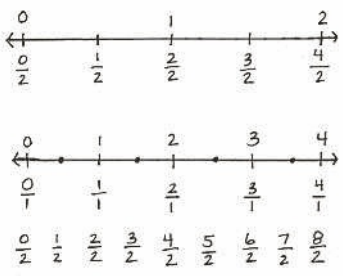
In this 35-day module, students extend and deepen Grade 2 practice with equal shares to understanding fractions as equal partitions of a whole (**2.3A**). Their knowledge becomes more formal as they work with area models and the number line. Throughout the module, students have multiple experiences working with the Grade 3 specified fractional units of halves, thirds, fourths, sixths, and eighths. To build flexible thinking about fractions, students are exposed to additional fractional units such as fifths, ninths, and tenths.

Topic A opens Module 5 with students actively partitioning different models of wholes into equal parts (e.g., concrete models and drawn pictorial area models on paper). They identify and count unit fractions as *1 half, 1 fourth, 1 third, 1 sixth*, and *1 eighth* in unit form and recognize that equal shares of identical wholes need not have the same shape (**3.6E**). In Topic B, students are introduced to the fraction form $\frac{1}{b}$ (**3.3C, 3.3D, 3.3E**) and understand that fractions are numbers. Just like any number, they can be written in different forms.

Students compare and make copies of unit fractions to build non-unit fractions. They understand unit fractions as the basic building blocks that compose other fractions (**3.3H**), which parallels the understanding that the number 1 is the basic building block of whole numbers (e.g., 1 and 1 and 1 make 3 just as 1 third and 1 third and 1 third make 1). Using this understanding, students write non-unit fractions as sums of unit fractions. In Topic C, students practice comparing unit fractions using fraction strips. They specify the whole and label fractions in relation to the number of equal parts in that whole (**3.3H**).



Compare unit fractions using fraction strips.



Students transfer their work to the number line in Topic D. They begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, count, and compare fractions on the number line (**3.3A, 3.3B, 3.3H, 3.7A**). In Topic E, they notice that some fractions with different units are placed at the exact same point on the number line, and therefore, are equal (**3.3F, 3.3G**). For example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are equivalent fractions (**3.3F**); they are different ways of naming the same number. Students recognize that whole numbers can be written as fractions, as exemplified on the number lines to the left (**3.3G**).

Topic F concludes the module with comparing fractions that have the same numerator. As students compare fractions by reasoning about their size, they understand that fractions with the same numerator and a larger denominator are actually smaller pieces of the whole (**3.3H**). Topic F leaves students with a new method for precisely partitioning a number line into unit fractions of any size without using a ruler.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Omit Lesson 3. Lesson 3's objective is similar to Lesson 2's. The difference is a shift from concrete to pictorial. Students will have exposure to extensive pictorial practice throughout the module.

Omit Lesson 4. Although Lesson 4 is an exploratory lesson that affords students the opportunity to synthesize their learning, no new material is presented.

Consolidate Lessons 11 and 12, both of which have nearly identical objectives and provide practice comparing unit fractions pictorially. Within the lesson that results, incorporate a variety of models into practice.

Omit Lesson 14. Lesson 14 provides practice with concepts and skills taught in the three preceding lessons. Although this lesson deepens practice, no new material is presented.

Omit Lesson 20. Lesson 20, designated as an optional lesson in the teaching sequence, provides practice with concepts and skills taught in the five preceding lessons.

Consider omitting Lesson 26 since its content is embedded into the work of prior lessons. Ensure that students have practiced counting and labeling whole number fractions as part of their work with fractions on the number line.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to represent and explain fractional units. The student is expected to:

3.3A represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines;

3.3B determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line;

3.3C explain that the unit fraction 1/b represents the quantity formed by one part of a whole that has been partitioned into b equal parts where b is a non-zero whole number;

3.3D compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts 1/b;

3.3E solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8;

3.3F represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;

3.3G explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model;

3.3H compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models.

Geometry and Measurement

The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:

3.6E decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape.

Geometry and Measurement

The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:

3.7A represent fractions of halves, fourths, and eighths as distances from zero on a number line.

Foundational Standards

The student is expected to:

2.3A partition objects into equal parts and name the parts, including halves, fourths, and eighths, using words;

2.3C use concrete models to count fractional parts beyond one whole using words and recognize how many parts it takes to equal one whole;

2.3D identify examples and non-examples of halves, fourths, and eighths;

2.8E decompose two-dimensional shapes such as cutting out a square from a rectangle, dividing a shape in half, or partitioning a rectangle into identical triangles and identify the resulting geometric parts;

2.9F use concrete models of square units to find the area of the rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
3.6E 3.3C 3.3D 3.3E	1.C 1.E 2.C 2.E 2.I 3.C 3.G 4.F 5.D 5.G	A	**Partitioning a Whole into Equal Parts** Lesson 1: Specify and partition a whole into equal parts, identifying and counting unit fractions using concrete models. Lesson 2: Specify and partition a whole into equal parts, identifying and counting unit fractions by folding fraction strips. Lesson 3: Specify and partition a whole into equal parts, identifying and counting unit fractions by drawing pictorial area models. Lesson 4: Represent and identify fractional parts of different wholes. Lesson 5: Recognize that equal parts of an identical rectangle can have different shapes.	5
3.3C **3.3D** **3.3E** 3.3G 3.6E	1.C 1.D 2.E 2.I 3.C 3.D 3.E 4.D 5.F	B	**Unit Fractions and Their Relation to the Whole** Lesson 6: Partition a whole into equal parts and define the equal parts to identify the unit fraction numerically. Lesson 7: Build non-unit fractions less than one whole from unit fractions. Lesson 8: Identify and represent shaded and non-shaded parts of one whole as fractions. Lesson 9: Represent parts of one whole as fractions with number bonds. Lesson 10: Build and write fractions greater than one whole using unit fractions.	5

TEKS	ELPS		Topics and Objectives	Days
3.3H 3.3C 3.3D 3.3E 3.3F 3.3G 3.6E	1.C 2.E 2.G 2.I 3.C 3.H 4.K 5.G	C	**Comparing Unit Fractions and Specifying the Whole** Lesson 11: Compare unit fractions by reasoning about their size using fraction strips. Lesson 12: Compare unit fractions with different-sized models representing the whole. Lesson 13: Specify the corresponding whole when presented with one equal part. Lesson 14: Identify a shaded fractional part in different ways depending on the designation of the whole.	4
			Mid-Module Assessment: Topics A–C (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
3.3A **3.3B** **3.3G** **3.3H** **3.7A**	1.A 1.C 2.D 2.E 2.F 2.H 2.I 3.D 4.G 5.G	D	**Fractions on the Number Line** Lesson 15: Place fractions on a number line with endpoints 0 and 1. Lesson 16: Place any fraction on a number line with endpoints 0 and 1. Lesson 17: Place whole number fractions and fractions between whole numbers on the number line. Lesson 18: Practice placing various fractions on the number line. Lesson 19: Compare fractions and whole numbers on the number line by reasoning about their distance from 0. Lesson 20: Understand distance and position on the number line as strategies for comparing fractions. (Optional)	6
3.3F **3.3G**	1.A 1.H 2.E 2.I 3.C 3.E 3.J 4.J 5.G	E	**Equivalent Fractions** Lesson 21: Recognize and show that equivalent fractions have the same size, though not necessarily the same shape. Lesson 22: Recognize and show that equivalent fractions refer to the same point on the number line. Lessons 23–24: Generate simple equivalent fractions by using visual fraction models and the number line. Lesson 25: Express whole numbers as fractions and recognize equivalence with different units.	8

TEKS	ELPS	Topics and Objectives		Days
			Lesson 26: Express whole number fractions on the number line when the unit interval is 1. Lesson 27: Decompose whole number fractions greater than 1 using whole number equivalence with various models. Lesson 28: Explain equivalence by manipulating units and reasoning about their size.	
3.3H	1.C 1.E 2.I 3.C 3.E 3.H 4.F 5.G	F	**Comparison, Order, and Size of Fractions** Lesson 29: Compare fractions with the same numerator pictorially. Lesson 30: Compare fractions with the same numerator using <, >, or =, and use a model to reason about their size. Lesson 31: Partition various wholes precisely into equal parts using a number line method.	3
			End-of-Module Assessment: Topics A–F (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
Total Number of Instructional Days				**35**

Terminology

New or Recently Introduced Terms

- Copies (refers to the number of unit fractions in 1 whole)
- Equivalent fractions (fractions that name the same size or the same point on the number line)
- Fraction form (e.g., $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$)
- Fractional unit (half, third, fourth, etc.)
- Non-unit fraction (fraction with numerator other than 1)
- Unit form (in reference to fractions, e.g., 1 half, 2 thirds, 4 fifths)
- Unit fraction (fraction with numerator 1)
- Unit interval (the interval from 0 to 1, measured by length)

Familiar Terms and Symbols[1]

- =, <, > (equal, less than, greater than)
- Array (arrangement of objects in rows and columns)
- Equal parts (parts with equal measurements)

[1] These are terms and symbols students have used or seen previously.

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GRADE 3 • MODULE 6

Financial Literacy and Data

Grade 3 • Module 6

Financial Literacy and Data

OVERVIEW

This 16-day module introduces topics of financial literacy and builds on Grade 2 concepts about data, graphing, and dot plots. Throughout the module, students are given repeated opportunities to expand upon familiar skills from Kindergarten, Grade 1, and Grade 2 as they engage in fluencies and application problems to count collections of bills and coins (**3.4C**).

In Topic A, students are introduced to a variety of concepts related to financial literacy. To begin, students research a variety of professions and identify the education requirements and the average salary associated with the different professions (**3.9A**). Students learn to analyze data as they explore the relationship between supply and demand as well as the benefits of saving money (**3.9B, 3.9C, 3.9E**). Finally, students are introduced to the concepts of credit, borrowing, and lending as students solve problems related to spending, saving, and borrowing (**3.9D, 3.9F**).

Topic B begins with a lesson in which students generate categorical data, organize it, and then represent it in a variety of forms. Drawing on Grade 2 knowledge, students might initially use tally marks, tables, or graphs with one-to-one correspondence. By the end of Lesson 5, they show data in strip diagrams where units are equal groups with a value greater than 1. In the next two lessons, students rotate the strip diagrams vertically so that the strips become the units or bars of scaled graphs (**3.8A, 3.8B**). Students understand picture and bar graphs as vertical representations of strip diagrams and apply well-practiced skip-counting and multiplication strategies to analyze them. In Lesson 8, students synthesize and apply learning from Topic B to solve one- and two-step problems. Through problem solving, opportunities naturally surface for students to make observations, analyze, and answer questions such as, “How many more?” or “How many less?” (**3.8A, 3.8B**).

In Topic C, students learn that intervals do not have to be whole numbers but can have fractional values that facilitate recording measurement data with greater precision. In Lesson 9, they generate a six-inch ruler marked in whole-inch, half-inch, and quarter-inch increments, using the Module 5 concept of partitioning a whole into parts. This creates a conceptual link between measurement and recent learning about fractions. Students then use the rulers to measure the lengths of precut straws and record their findings to generate measurement data (**3.7A**).

Lesson 12 reintroduces dot plots as a tool for displaying measurement data. Although familiar from Grade 2, dot plots in Grade 3 have the added complexity of including fractions on the number line (**2.9A, 3.7A**). In this lesson, students interpret scales involving whole, half, and quarter units in order to analyze data. This experience lays the foundation for them to create their own dot plots in Lessons 13 and 14. To draw dot plots, students learn to choose appropriate intervals within which to display a particular set of data. For example, to show measurements of classmates’ heights, students might notice that their data fall within the range of 45 to 55 inches and then construct a dot plot with the corresponding interval.

Students end the module by applying learning from Lessons 5–14 to problem solving. They work with a mixture of scaled picture graphs, bar graphs, and dot plots to problem solve using both categorical and measurement data (**3.7A, 3.8A, 3.8B**).

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Counting collections of bills and coins (**3.4C**) is covered using the fluency activity titled *Name the Value* throughout the module. It is found in Lessons 1, 2, 7, 9, and 11. If modifications are deemed necessary to the listed lessons, be sure to embed these fluencies at other points in the module. Counting collections of bills and coins is also found in the Application Problems in Lessons 5, 10, 12, and 15. These problems are important to include, should other modifications be made to these lessons.

Omit Lesson 15, a problem solving lesson involving categorical and measurement data. Be sure to embed problem solving practice with both types of data into prior lessons.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

3.4C determine the value of a collection of coins and bills.

Geometry and Measurement

The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:

3.7A represent fractions of halves, fourths, and eighths as distances from zero on a number line;

3.7D determine when it is appropriate to use measurements of liquid volume (capacity) or weight;

3.7E determine liquid volume (capacity) or weight using appropriate units and tools.

Data Analysis

The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

3.8A summarize a data set with multiple categories using a frequency table, dot plot, pictograph, or bar graph with scaled intervals;

3.8B solve one- and two-step problems using categorical data represented with a frequency table, dot plot, pictograph, or bar graph with scaled intervals.

Personal Financial Literacy

The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

3.9A explain the connection between human capital/labor and income;

3.9B describe the relationship between the availability or scarcity of resources and how that impacts cost;

3.9C identify the costs and benefits of planned and unplanned spending decisions;

3.9D explain that credit is used when wants or needs exceed the ability to pay and that it is the borrower's responsibility to pay it back to the lender, usually with interest;

3.9E list reasons to save and explain the benefit of a savings plan, including for college;

3.9F identify decisions involving income, spending, saving, credit, and charitable giving.

Foundational Standards

The student is expected to:

2.2E locate the position of a given whole number on an open number line;

2.2F name the whole number that corresponds to a specific point on a number line;

2.5A determine the value of a collection of coins up to one dollar;

2.5B use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins;

2.9A find the length of objects using concrete models for standard units of length;

2.9C represent whole numbers as distances from any given location on a number line;

2.9E determine a solution to a problem involving length, including estimating lengths;

2.10A explain that the length of a bar in a bar graph or the number of pictures in a pictograph represents the number of data points for a given category;

2.10B organize a collection of data with up to four categories using pictographs and bar graphs with intervals of one or more;

2.10C write and solve one-step word problems involving addition or subtraction using data represented with in pictograph or bar graphs with intervals of one;

2.11B explain that saving is an alternative to spending;

2.11D identify examples of borrowing and distinguish between responsible and irresponsible borrowing;

2.11E identify examples of lending and use concepts of benefits and costs to evaluate lending decisions.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

MPS(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS		Topics and Objectives	Days
3.9A **3.9B** **3.9C** **3.9D** **3.9E** **3.9F** 3.4C	1.A 1.F 2.C 3.C 3.E 4.E 4.I 4.J 5.G	A	**Financial Literacy** Lesson 1: Explore the relationship between human capital and income by generating income data for various professions. Lesson 2: Explore relationships between availability of resources and cost. Lesson 3: Recognize the benefits of saving. Lesson 4: Understand credit and the role of borrower and lender.	4
3.8A **3.8B** 3.4C	1.A 1.C 2.C 2.E 3.E 3.F 4.I 5.F	B	**Generate and Analyze Categorical Data** Lesson 5: Generate and organize data. Lesson 6: Rotate strip diagrams vertically. Lesson 7: Create scaled bar graphs. Lesson 8: Solve one- and two-step problems involving graphs.	4
3.7A **3.7D** **3.7E** 3.4C	2.C 2.E 2.I 3.C 3.D 3.E 3.J 4.I 5.G	C	**Generate and Analyze Measurement Data** Lesson 9: Create a ruler with 1-inch, $\frac{1}{2}$-inch, and $\frac{1}{4}$-inch intervals, and generate measurement data. Lesson 10: Explore customary weight units and generate measurement data. Lesson 11: Explore customary capacity units and generate measurement data. Lesson 12: Interpret measurement data from various dot plots. Lessons 13–14: Represent measurement data with dot plots. Lesson 15: Analyze data to problem solve.	7
			End-of-Module Assessment: Topics A–C (assessment ½ day, return and remediation ½ day)	1
Total Number of Instructional Days				**16**

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GRADE 3 • MODULE 7

Geometry and Measurement Word Problems

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Geometry and Measurement Word Problems

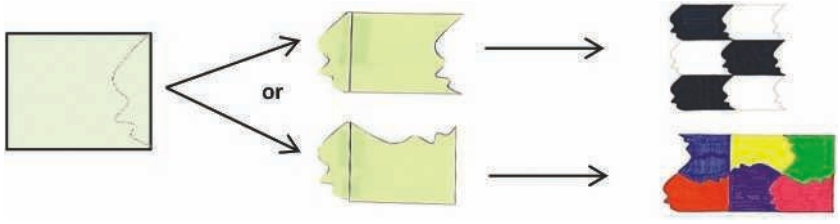
OVERVIEW

The final module of the year offers students intensive practice with word problems, as well as hands-on investigation experiences with geometry and perimeter.

Topic A begins with solving one- and two-step word problems based on a variety of topics studied throughout the year and including all four operations (**3.4K, 3.5A, 3.5B**). The lessons emphasize modeling and reasoning to develop solution paths. They incorporate teacher-facilitated problem solving, opportunities for students to independently make sense of problems and persevere in solving them, and time for students to share solutions and critique peer strategies.

Topic B introduces an exploration of geometry. Students build on Grade 2 ideas about polygons and their properties, specifically developing and expanding their knowledge of quadrilaterals. They explore the attributes of quadrilaterals and classify examples into various categories, including recognizing the characteristics of polygons (**3.6B**). Students draw polygons based on their attributes, producing sketches from descriptions like, "This shape has two long sides that are parallel, two short sides, and no right angles." Student classify and sort three-dimensional solids and figures in the final two lessons of Topic B (**3.6A**).

Students tessellate to bridge geometry experience with the study of perimeter in Topic C. They first decompose a quadrilateral and then rearrange the parts. They use the new shape to tile. Students then define perimeter in two distinct ways: (1) as the boundary of a planar region and (2) as the length of the boundary curve. Students see varied examples from the tiles used to tessellate.



Cut on the line. Then, slide the piece to the opposite side or rotate it to an adjacent side to make a new shape. Then, tile with the new shape.

As they learn about perimeter as an attribute of plane figures, students apply their knowledge to real-world situations through problem solving (**3.7B**). They measure side lengths of shapes in whole number units to determine perimeter and solve problems where side lengths are given. They use string and rulers to measure the length around circles of different sizes. This variation prompts students to think more flexibly about

perimeter, understanding that it can be the boundary of any shape and that its measurements are not limited to whole numbers. The topic ends with problems in which some measurements around the perimeter of a polygon are unknown but can be determined by reasoning. Students consider the efficiency of their strategies and identify tools for solving; for example, they use multiplication as a tool when measurements are repeated.

Topic D utilizes the dot plot, familiar from Module 6, to help students draw conclusions about perimeter and area measurements (**3.7A**). Early in the topic, students find different possible perimeters or areas for rectangles based on information given about the rectangles. For example, using knowledge of factors from experience with multiplication, students find the following:

- Different perimeters of rectangles composed of a given number of unit squares (**3.7B**).
  For example, given a rectangle composed of 24 unit squares, students find four possible perimeters: 50, 28, 22, and 20 length units.
- Different areas of rectangles with a given perimeter and composed of unit squares.
  For example, students use unit squares to build rectangles with a perimeter of 12 units and determine that they can do so using 5, 8, or 9 unit squares.

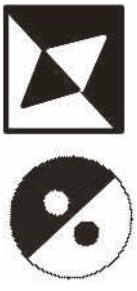
(Forming rectangles with unit squares results in whole number side lengths.)

Students use dot plots to show the number of rectangles they were able to construct for each set of given information. The dot plots are tools that students use to help them analyze and draw conclusions about their data. Students draw their rectangles on grid paper and reason about their findings. They notice, for example, that for rectangles of a given area, those with side lengths that are equal or almost equal (more square-like) have smaller perimeters than those whose side lengths are very different (a long and narrow shape).

By the end of the topic, students are able to conclude that there is no direct relationship between area and perimeter. If an area is given, there is no way of knowing a shape's corresponding perimeter without more information about the side lengths.

In Topic E, students synthesize their learning in the final lessons through solving word problems involving area and perimeter using all four operations (**3.5A, 3.5B, 3.6C, 3.6D, 3.7B**).

Topic F concludes the school year with a set of engaging lessons that briefly review the fundamental Grade 3 concepts of fractions, multiplication, and division. This topic comes after the End-of-Module Assessment. It begins with a pair of lessons on fractions, engaging students in analyzing and creating unusual representations of one-half, such as those shown to the right. Students analyze and discuss these representations, using their knowledge of fractions to justify their constructions and critique the work of others. The final lessons in this topic are fluency based and engage students in games that provide practice to solidify their automaticity with Grade 3 skills. Using simple origami techniques, students create booklets of these games. The booklets go home and become resources for summer practice.



Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Omit Lesson 10. Tessellating helps students understand that perimeter is not just a property of shapes with straight sides. Lesson 15 revisits this idea.

Omit Lesson 21. This lesson culminates Topic D by having students record data collected from Lessons 19 and 20 on a dot plot. Although it deepens understanding of concepts explored in earlier lessons, no new material is presented.

Omit Lessons 25–28, which provide a review of important Grade 3 material including fluency and fractions. Be sure, however, to notice the resources for summer practice included in Lesson 28.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

Algebraic Reasoning

The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

3.5A represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;

3.5B represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations.

Geometry and Measurement

The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:

3.6A classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language;

3.6B use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories.

Geometry and Measurement

The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:

3.7A represent fractions of halves, fourths, and eighths as distances from zero on a number line;

3.7B determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in problems.

Foundational Standards

The student is expected to:

2.2E locate the position of a given whole number on an open number line;

2.2F name the whole number that corresponds to a specific point on a number line;

2.8A create two-dimensional shapes based on given attributes, including number of sides and vertices;

2.8B classify and sort three-dimensional solids including spheres, cones, cylinders, rectangular prisms (including cubes as special rectangular prisms) and triangular prisms, based on attributes using formal geometric language;

2.8C classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices;

2.8D compose two-dimensional shapes and three-dimensional solids with given properties or attributes;

2.9A find the length of objects using concrete models for standard units of length;

2.9C represent whole numbers as distances from any given location on a number line;

2.9D determine the length of an object to the nearest marked unit using rulers, yardsticks, meter sticks, or measuring tapes;

3.6C determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;

3.6D decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(A) apply mathematics to problems arising in everyday life, society, and the workplace;

MPS(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

MPS(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
3.4K **3.5A** **3.5B**	1.F 2.E 3.E 4.G 5.G	A	**Solving Word Problems** Lessons 1–2: Solve word problems in varied contexts. Lesson 3: Share and critique peer solution strategies to varied word problems.	3
3.6A **3.6B**	1.D 1.E 2.B 2.E 3.E 3.J 4.B 5.G	B	**Attributes of Two- and Three-Dimensional Figures** Lesson 4: Compare and classify quadrilaterals. Lesson 5: Compare and classify other polygons. Lesson 6: Draw polygons with specified attributes to solve problems. Lessons 7–8: Classify and sort three-dimensional figures according to their attributes.	5

TEKS	ELPS	Topics and Objectives		Days
3.7B 3.5A 3.5B 3.6B	1.C 2.C 2.E 2.G 2.I 3.A 3.E 3.H 4.J 5.G	C	**Problem Solving with Perimeter** Lesson 9: Decompose quadrilaterals to understand perimeter as the boundary of a shape. Lesson 10: Tessellate to understand perimeter as the boundary of a shape. (Optional) Lesson 11: Measure side lengths in whole number units to determine the perimeter of polygons. Lesson 12: Explore perimeter as an attribute of plane figures and solve problems. Lesson 13: Determine the perimeter of regular polygons and rectangles when whole number measurements are unknown. Lesson 14: Solve word problems to determine perimeter with given side lengths. Lesson 15: Use string to measure the perimeter of various circles to the nearest quarter inch. Lesson 16: Use all four operations to solve problems involving perimeter and unknown measurements.	8
			Mid-Module Assessment: Topics A–C (assessment 1 day, return 1 day, remediation or further applications 1 day)	3
3.7A **3.7B** 3.6A 3.6B	1.C 2.E 3.C 3.E 3.H 3.J 4.J 5.B	D	**Recording Perimeter and Area Data on Dot Plots** Lesson 17: Construct rectangles from a given number of unit squares and determine the perimeters. Lesson 18: Use a dot plot to record the number of rectangles constructed from a given number of unit squares. Lessons 19–20: Construct rectangles with a given perimeter using unit squares and determine their areas. Lesson 21: Use a dot plot to record the number of rectangles constructed in Lessons 19 and 20.	5

TEKS	ELPS		Topics and Objectives	Days
3.6A **3.6B** **3.7B** 3.4K 3.5A 3.5B	1.C 2.E 3.G 3.H 4.G	E	**Problem Solving with Perimeter and Area** Lesson 22: Solve a variety of word problems with perimeter. Lessons 23–24: Solve a variety of word problems involving area and perimeter using all four operations.	3
			End-of-Module Assessment: Topics A–E (assessment 1 day, return $\frac{1}{2}$ day, remediation or further applications $\frac{1}{2}$ day)	2
*		F	**Year in Review** Lessons 25–26: Explore and create unconventional representations of one-half. Lesson 27: Solidify fluency with Grade 3 skills. Lesson 28: Create resource booklets to support fluency with Grade 3 skills.	4
Total Number of Instructional Days				**33**

**The Year in Review offers a spiral review of the major work of the current grade level with a view toward success in the succeeding grade level.*