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GRADE 4 • MODULE 1

Place Value, Rounding, and Algorithms for Addition and Subtraction

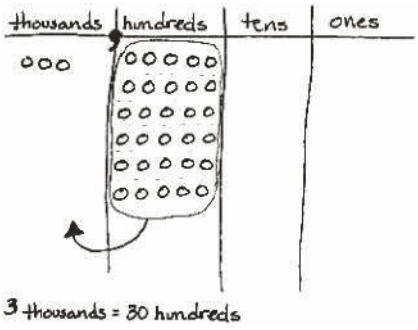
Grade 4 • Module 1

Place Value, Rounding, and Algorithms for Addition and Subtraction

OVERVIEW

In this 25-day Grade 4 module, students extend their work with whole numbers. They begin with large numbers using familiar units (hundreds and thousands) and develop their understanding of billions by building knowledge of the pattern of *times ten* in the base ten system on the place value chart (**4.2A**).[1] They recognize that each sequence of three digits is read as hundreds, tens, and ones followed by the naming of the corresponding base thousand unit (thousand, million, billion).

The place value chart is fundamental to Topic A. Building upon their previous knowledge of bundling, students learn that 10 hundreds can be composed into 1 thousand, and therefore, 30 hundreds can be composed into 3 thousands because a digit's value is 10 times what it would be one place to its right (**4.2A**). Students learn to recognize that in a number such as 7,777, each 7 has a value that is 10 times the value of its neighbor to the immediate right. One thousand can be decomposed into 10 hundreds; therefore 7 thousands can be decomposed into 70 hundreds.



Similarly, multiplying by 10 shifts digits one place to the left, and dividing by 10 shifts digits one place to the right.

$$3{,}000 = 10 \times 300 \qquad 3{,}000 \div 10 = 300$$

[1] Grade 4 students will explore the pattern of *one-tenth as much* in Module 6.

In Topic B, students use place value as a basis for comparing whole numbers. Although this is not a new concept, it becomes more complex as the numbers become larger. For example, it becomes clear that 34,156 is 3 thousands greater than 31,156.

$$34{,}156 > 31{,}156$$

Comparison leads directly into rounding, where their skill with isolating units is applied and extended. Rounding to the nearest ten and hundred was mastered with three-digit numbers in Grade 3. Now, Grade 4 students moving into Topic C learn to round through the hundred thousands place (**4.2D**), initially using the vertical number line though ultimately moving away from the visual model altogether. Topic C also includes word problems where students apply rounding to real life situations.

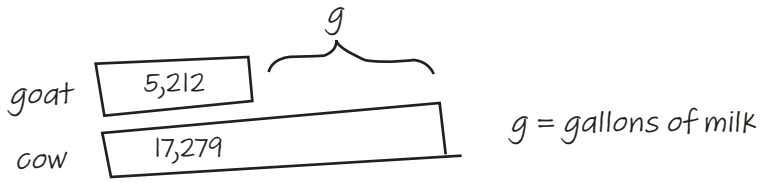
In Grade 4, students become fluent with the standard algorithms for addition and subtraction. In Topics D and E, students focus on single like-unit calculations (ones with ones, thousands with thousands, etc.), at times requiring the composition of greater units when adding (10 hundreds are composed into 1 thousand) and decomposition into smaller units when subtracting (1 thousand is decomposed into 10 hundreds) (**4.4A**). Throughout these topics, students apply their algorithmic knowledge to solve word problems. Students also use a letter to represent the unknown quantity.

The module culminates with multi-step word problems in Topic F (**4.5A**). Strip diagrams are used throughout the topic to model *additive compare* problems like the one exemplified below. These diagrams facilitate deeper comprehension and serve as a way to support the reasonableness of an answer. TEKS standard 4.5A will continue to be developed throughout the course of Grade 4 through many exposures and in many contexts.

A goat produces 5,212 gallons of milk a year.

A cow produces 17,279 gallons of milk a year.

How much more milk does a goat need to produce to make the same amount of milk as a cow?



17,279 – 5,212 = g

A goat needs to produce 12,067 *more gallons of milk a year.*

The Mid-Module Assessment follows Topic C. The End-of-Module Assessment follows Topic F.

Notes on Pacing and Differentiation

If pacing is a challenge, consider omitting Lesson 17 since multi-step problems are taught in Lesson 18. Instead, embed problems from Lesson 17 into Module 2 or 3 as extensions. Since multi-step problems are taught in Lesson 18, Lesson 19 could also be omitted.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:

4.2A interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;[2]

4.2B represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals;

4.2C compare and order whole numbers to 1,000,000,000 and represent comparisons using the symbols >, <, or =;

4.2D round whole numbers to a given place value through the hundred thousands place.

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:

4.4A add and subtract whole numbers and decimals to the hundredths place using the standard algorithm;

4.4G round to the nearest 10, 100, or 1,000 or use compatible numbers to estimate solutions involving whole numbers.

Algebraic Reasoning

The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

4.5A represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity.[3]

[2] Grade 4 students will explore the pattern of *one-tenth as much* in Module 6.

[3] The remainder of 4.5A will be covered in Module 3 and throughout all other modules through Application Problems.

Foundational Standards

The student is expected to:

3.2B describe the mathematical relationships found in the base-10 place value system through the hundred thousands place;

3.2C represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words to describe relative size of numbers in order to round whole numbers;

3.2D compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =;

3.4A solve with fluency one-step and two-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction;

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts;

3.5A represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

MPS(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS		Topics and Objectives	Days
4.2A **4.2B** 4.2C 3.5C	1.C 1.F 2.B 2.E 2.I 3.B 3.E 3.G 3.I 4.G 5.B	A	**Place Value of Multi-Digit Whole Numbers** Lesson 1: Interpret a multiplication equation as a comparison. Lesson 2: Recognize a digit represents 10 times the value of what it represents in the place to its right. Lesson 3: Name numbers within 1 billion by building understanding of the place value chart and placement of commas for naming base thousand units. Lesson 4: Read and write multi-digit numbers using base ten numerals, number names, and expanded form.	4
4.2C 4.2B	1.C 2.C 2.I 3.E 4.D 4.G 5.B	B	**Comparing Multi-Digit Whole Numbers** Lesson 5: Compare numbers based on meanings of the digits using >, <, or = to record the comparison. Lesson 6: Find 1, 10, and 100 thousand and 1, 10, and 100 million more and less than a given number.	2
4.2D **4.4G**	1.A 1.C 2.C 2.I 3.D 3.E 3.G 4.J 5.B	C	**Rounding Multi-Digit Whole Numbers** Lesson 7: Round multi-digit numbers to the thousands place using the vertical number line. Lesson 8: Round multi-digit numbers to any place using the vertical number line. Lesson 9: Use place value understanding to round multi-digit numbers to any place value. Lesson 10: Use place value understanding to round multi-digit numbers to any place value using real world applications.	4
			Mid-Module Assessment: Topics A–C (review content 1 day, assessment ½ day, return ½ day, remediation or further applications 1 day)	3

TEKS	ELPS	Topics and Objectives		Days
4.4A **4.4G** **4.5A** 4.2A 4.2B 4.2C	1.C 2.G 2.I 3.E 4.G 4.J 5.A 5.B 5.D 5.E	D	**Multi-Digit Whole Number Addition** Lesson 11: Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using strip diagrams. Lesson 12: Solve multi-step word problems using the standard addition algorithm modeled with strip diagrams, and assess the reasonableness of answers using rounding.	2
4.4A **4.4G** **4.5A** 4.2A 4.2B	1.C 1.H 2.E 2.I 3.E 3.F 4.G 4.J 5.B	E	**Multi-Digit Whole Number Subtraction** Lesson 13: Use place value understanding to decompose to smaller units once using the standard subtraction algorithm, and apply the algorithm to solve word problems using strip diagrams. Lesson 14: Use place value understanding to decompose to smaller units up to three times using the standard subtraction algorithm, and apply the algorithm to solve word problems using strip diagrams. Lesson 15: Use place value understanding to fluently decompose to smaller units multiple times in any place using the standard subtraction algorithm, and apply the algorithm to solve word problems using strip diagrams. Lesson 16: Solve two-step word problems using the standard subtraction algorithm fluently modeled with strip diagrams, and assess the reasonableness of answers using rounding.	4
4.5A 4.2A 4.2B 4.2C 4.4A	1.B 1.C 2.I 3.E 3.G 3.H 3.I 4.G 4.J 5.B	F	**Addition and Subtraction Word Problems** Lesson 17: Solve *additive compare* word problems modeled with strip diagrams. Lesson 18: Solve multi-step word problems modeled with strip diagrams, and assess the reasonableness of answers using rounding. Lesson 19: Create and solve multi-step word problems from given strip diagrams and equations.	3
			End-of-Module Assessment: Topics A–F (review content 1 day, assessment ½ day, return ½ day, remediation or further application 1 day)	3
Total Number of Instructional Days				**25**

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GRADE 4 • MODULE 2

Unit Conversions and Problem Solving with Metric Measurements

Grade 4 • Module 2

Unit Conversions and Problem Solving with Metric Measurement

OVERVIEW

The idea of a mixed unit shows up in varied contexts. For instance, students have become accustomed to thinking of 250 as the mixed units of 2 hundreds 5 tens. Mixed units are also used in the context of 2 hr 5 min, \$2.50, 2 km 5 m, 2' 5", and $2\frac{5}{8}$ (hours and minutes, dollars and cents, kilometers and meters, feet and inches, ones and eighths). While the context and the units may vary greatly, there are many common threads present in any mixed unit calculation. Consider the connections and similarities between the following equalities:

2 thousands	437 ones	=	2,437 ones
2 kilometers	437 meters	=	2,437 meters
2 kilograms	437 grams	=	2,437 grams
2 liters	437 milliliters	=	2,437 milliliters

In order to explore the process of working with mixed units, Module 2 focuses on length, mass, and capacity in the metric system where place value serves as a natural guide for moving between larger and smaller units. Conversions of smaller units to larger units as well as conversions within the customary measurement system will be explored in Module 7.

In Topic A, students review place value concepts while building fluency with decomposing, or converting from larger to smaller units (**4.8A, 4.8B**). They learn the relative sizes of measurement units, building off prior knowledge of grams and kilograms from Grade 3 (**3.7E**) and meters and centimeters from Grade 2 (**2.9E**). Conversions between the units are recorded in a two-column table. Single-step problems involving addition and subtraction of metric units provide an opportunity to practice mental math calculations as well as the addition and subtraction algorithms established in Module 1. Students reason by choosing to convert between mixed and single units before or after the computation (**4.8C**). Connecting their familiarity with both metric units and place value, the module moves swiftly through each unit of conversion, spending only one day on each type. This initial understanding of unit conversions allows for further application and practice, such as multiplying and dividing metric units, throughout subsequent modules.

In Topic B, students continue to build off their measurement work from previous grade levels. They solidify their understanding of the relationship between metric units and the place value chart and apply unit conversions to solve and reason about multi-step word problems (**4.8C**). Applying the skills learned in Module 1, students discover and explore the relationship between place value and conversions. The beauty of both the place value and measurement systems is the efficiency and precision permitted by the use of different size units to express a given quantity. As students solve word problems by adding and subtracting metric units, their ability to reason in parts and wholes is taken to the next level. This is important preparation

for multi-digit operations and for manipulating fractional units in future modules. Strip diagrams and number lines serve as models throughout the module to support the application of the standard algorithm to word problems.

Notes on Pacing for Differentiation

Although composed of just five lessons, Module 2 has great importance in the Grade 4 sequence of modules. Module 2, along with Module 1, is paramount in setting the foundation for developing fluency with the manipulation of place value units, a skill upon which Module 3 greatly depends. Teachers who have taught Module 2 prior to Module 3 have reportedly moved through Module 3 more efficiently than colleagues who have omitted it. Module 2 also sets the foundation for work with fractions and mixed numbers in Module 5. Therefore, it is not recommended to omit any lessons from Module 2.

To help with the pacing of Module 3's Topic A, consider replacing the Convert Units fluencies in Module 2, Lessons 1–3, with area and perimeter fluencies. Also, consider incorporating Problem 1 from Module 3, Lesson 1, into the fluency component of Module 2, Lessons 4 and 5.

Focus Grade Level Standards

Geometry and Measurement

The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:

4.8A identify relative sizes of measurement units within the customary and metric systems;

4.8B convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table;

4.8C solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.

Foundational Standards

The student is expected to:

2.2A use concrete and pictorial models to compose and decompose numbers up to 1,200 in more than one way as a sum of so many thousands, hundreds, tens, and ones;

2.9E determine a solution to a problem involving length, including estimating lengths;

3.7E determine liquid volume (capacity) or weight using appropriate units and tools;

4.4G round to the nearest 10, 100, or 1,000 or use compatible numbers to estimate solutions involving whole numbers;

4.4H solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders;

4.5A represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(A) apply mathematics to problems arising in everyday life, society, and the workplace;

MPS(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
4.8A **4.8B** **4.8C**	1.C 1.E 2.I 3.C 3.E 3.G 4.G 4.J 5.B	A	**Metric Unit Conversions** Lesson 1: Express metric length measurements in terms of a smaller unit; model and solve addition and subtraction word problems involving metric length. Lesson 2: Express metric mass measurements in terms of a smaller unit; model and solve addition and subtraction word problems involving metric mass. Lesson 3: Express metric capacity measurements in terms of a smaller unit; model and solve addition and subtraction word problems involving metric capacity.	3
4.8A **4.8B** **4.8C**	1.C 2.I 3.E 3.G 3.H 4.G 5.B	B	**Application of Metric Unit Conversions** Lesson 4: Know and relate metric units to place value units in order to express measurements in different units. Lesson 5: Use addition and subtraction to solve multi-step word problems involving length, mass, and capacity.	2
			End-of-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
Total Number of Instructional Days				**7**

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GRADE 4 • MODULE 3

Multi-Digit Multiplication and Division

Grade 4 • Module 3

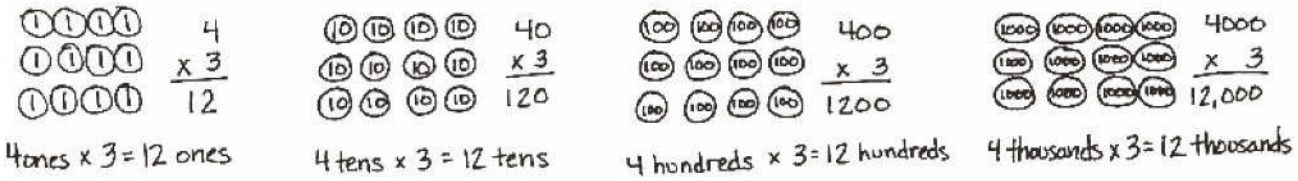
Multi-Digit Multiplication and Division

OVERVIEW

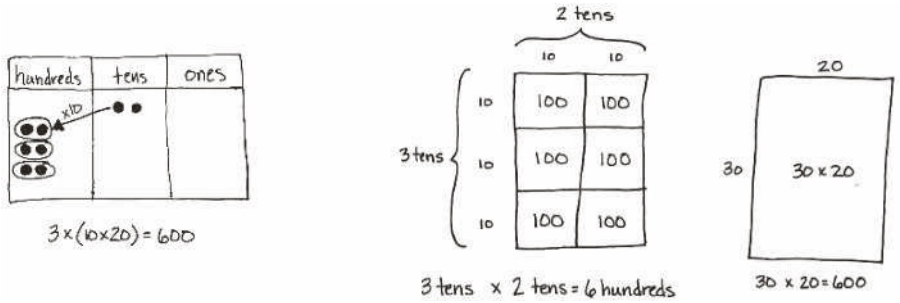
In this 39-day module, students use place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. As a key area of focus for Grade 4, this module moves slowly but comprehensively to develop students' ability to reason about the methods and models chosen to solve problems with multi-digit factors and dividends.

Students begin in Topic A by investigating the formulas for area and perimeter. They then solve multiplicative comparison problems using their Grade 3 knowledge and the language of "times as much" (**3.5C**) with a focus on problems using area and perimeter as a context (e.g., "A field is 9 feet wide. It is 4 times as long as it is wide. What is the perimeter of the field?"). Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities (**4.5A**). This is foundational for understanding multiplication as scaling and proportional reasoning in Grade 6. This Grade 4 module, beginning with area and perimeter, allows for new and interesting word problems as students learn to calculate with larger numbers and interpret more complex problems (**4.5C, 4.5D**).

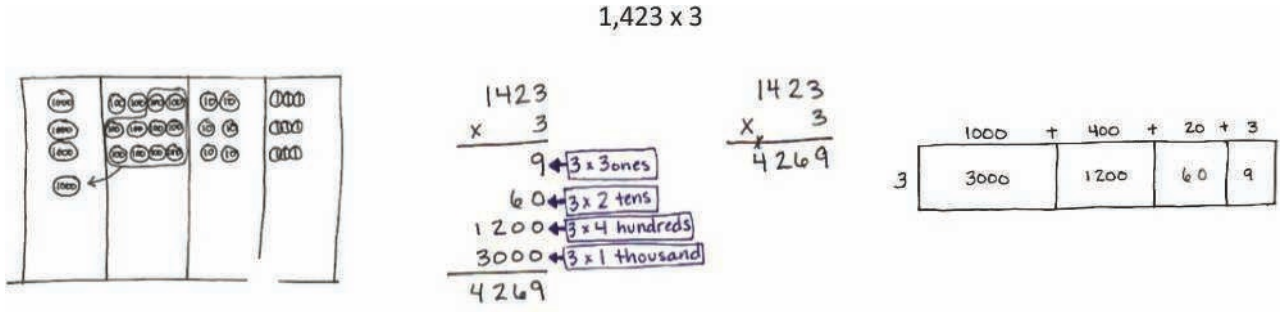
In Topic B, students use place value disks to multiply single-digit numbers by multiples of 10, 100, and 1,000 and two-digit multiples of 10 by two-digit multiples of 10 (**4.4C, 4.4D**). Reasoning between arrays and written numerical work allows students to see the role of place value units in multiplication (as pictured below). Students also practice the language of units to prepare them for multiplication of a single-digit factor by a factor with up to four digits and multiplication of two two-digit factors.



In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying two two-digit multiples of 10. For example, students have multiplied 20 by 10 on the place value chart and know that it shifts the value one place to the left, 10 × 20 = 200. To multiply 20 by 30, the associative property allows for simply tripling the product, 3 × (10 × 20), or multiplying the units, 3 tens × 2 tens = 6 hundreds (alternatively, (3 × 10) × (2 × 10) = (3 × 2) × (10 × 10)). Introducing this early in the module allows students to practice during fluency so that, by the time it is embedded within the two-digit by two-digit multiplication in Topic G, understanding and skill are in place.

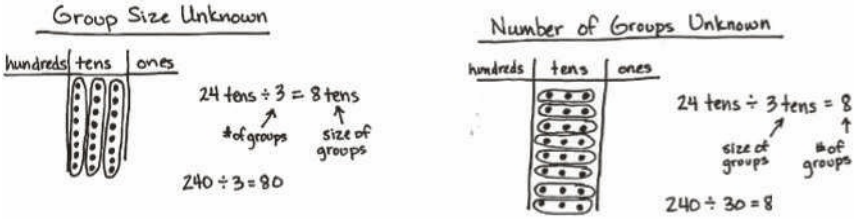


Building on their work in Topic B, students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students use the distributive property and multiply using place value disks to model. Practice with place value disks is used for two-, three-, and four-digit by one-digit multiplication problems with recordings as partial products. Students bridge partial products to the recording of multiplication via the standard algorithm.[1] Finally, the partial products method, the standard algorithm, and the area model are compared and connected by the distributive property (**4.4C, 4.4D**).



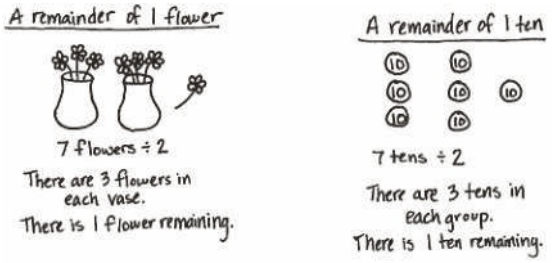
Topic D gives students the opportunity to apply their new multiplication skills to solve multi-step word problems (**4.4C, 4.4D**) and multiplicative comparison problems (**4.4H**). Students write equations from statements within the problems (**4.5A**) and use a combination of addition, subtraction, and multiplication to solve.

In Topic E, students synthesize their Grade 3 knowledge of division types (*group size unknown* and *number of groups unknown)* with their new, deeper understanding of place value.

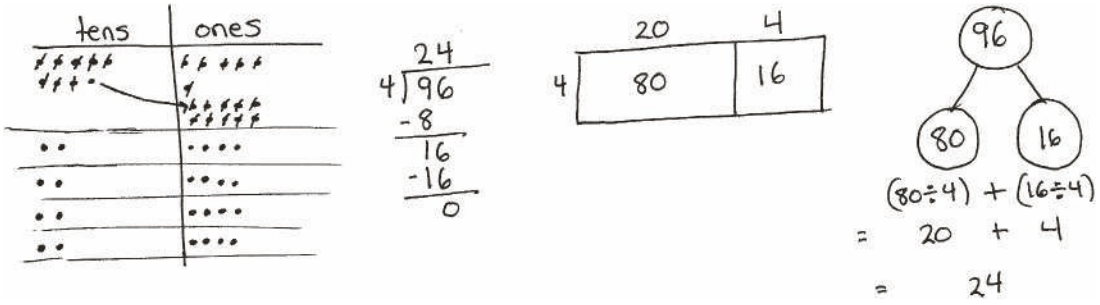


[1]Students become fluent with the standard algorithm for multiplication in Grade 5 (5.3B). Grade 4 students are introduced to the standard algorithm in preparation for fluency and as a general method for solving multiplication problems based on place value strategies, alongside place value disks, partial products, and the area model. Students are not assessed on the standard algorithm in Grade 4.

Students focus on interpreting the remainder within division problems, both in word problems and long division (**4.4H, 4.5A**). A remainder of 1, as exemplified below, represents a leftover flower in the first situation and a remainder of 1 ten in the second situation.[2]



While we have no reason to subdivide a remaining flower, there are good reasons to subdivide a remaining ten. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided. Students represent division with single-digit divisors using arrays and the area model before practicing with place value disks. The standard division algorithm[3] is practiced using place value knowledge, decomposing unit by unit. Finally, students use the area model to solve division problems, first with and then without remainders (**4.4E, 4.4F**).

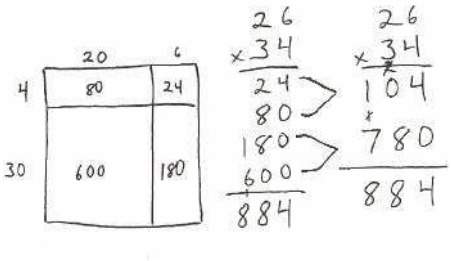


Topic F extends the practice of division with three- and four-digit dividends using place value understanding. A connection to Topic B is made initially with dividing multiples of 10, 100, and 1,000 by single-digit numbers. Place value disks support students visually as they decompose each unit before dividing. Students then practice using the standard algorithm to record long division. They solve word problems and make connections to the area model as was done with two-digit dividends (**4.4E, 4.4H, 4.5A**).

[2]Note that care must be taken in the interpretation of remainders. Consider the fact that 7 ÷ 3 is not equal to 5 ÷ 2 because the remainder of 1 is in reference to a different whole amount ($2\frac{1}{3}$ is not equal to $2\frac{1}{2}$).

[3]Students become fluent with the standard division algorithm in Grade 5 (5.3C, 5.3G). For adequate practice in reaching fluency, students are introduced to the division algorithm in Grade 4 as a general method for solving division problems.

The module closes as students multiply two-digit by two-digit numbers. Students use their place value understanding and understanding of the area model to empower them to multiply by larger numbers (as pictured to the right). Topic G culminates at the most abstract level by explicitly connecting the partial products appearing in the area model to the distributive property and recording the calculation vertically (**4.4C**). Students see that partial products written vertically are the same as those obtained via the distributive property: 4 twenty-sixes + 30 twenty-sixes = 104 + 780 = 884.



As students progress through this module, they are able to apply the multiplication and division algorithms because of their in-depth experience with the place value system and multiple conceptual models. This helps to prepare them for fluency with the multiplication and division algorithms in Grade 5. Students are encouraged in Grade 4 to continue using models to solve when appropriate.

Notes on Pacing for Differentiation

Within this module, if pacing is a challenge, consider the following omissions. In Lesson 1, omit Problem 1 if you embedded it into Module 2, and omit Problem 4, which can be used for a center activity. In Lesson 8, omit the drawing of models in Problems 2 and 4 of the Concept Development and in Problem 2 of the Problem Set. Instead, have students think about and visualize what they would draw. Omit Lesson 10 because the objective for Lesson 10 is the same as that for Lesson 9. Omit Lesson 19, and instead, embed discussions of interpreting remainders into other division lessons. Omit Lesson 21 because students solve division problems using the area model in Lesson 20. Omit Lesson 27, and instead, embed analysis of division situations throughout later lessons. Omit Lesson 29, and embed into Lesson 26 the discussion of the connection between division using the area model and division using the algorithm.

Look ahead to the Pacing Suggestions for Module 4. Consider partnering with the art teacher to teach Module 4's Topic A simultaneously with Module 3.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy.[4] The student is expected to:

4.4C represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15;

4.4D use strategies and algorithms, including the standard algorithm, to multiply up to a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

4.4E represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;

4.4F use strategies and algorithms, including the standard algorithm, to divide up to a four-digit dividend by a one-digit divisor;

4.4H solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders.

Algebraic Reasoning

The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

4.5A represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;

4.5C use models to determine the formulas for the perimeter of a rectangle (l + w + l + w or 2l + 2w), including the special form for perimeter of a square (4s) and the area of a rectangle (l x w);

4.5D solve problems related to perimeter and area of rectangles where dimensions are whole numbers.

[4] 4.4A is addressed in Module 1 and is then reinforced throughout the year.

Foundational Standards

The student is expected to:

3.4E represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;

3.4F recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;

3.4G use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts;

3.5A represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;

3.5B represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;

3.5C describe a multiplication expression as a comparison such as 3 x 24 represents 3 times as much as 24;

3.5D determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product;

3.6C determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;

3.6D decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
4.4H **4.5A** **4.5C** **4.5D** 4.4G	1.C 2.B 2.C 2.E 2.I 3.D 3.I 4.G 5.B	A	**Multiplicative Comparison Word Problems** Lesson 1: Investigate and use the formulas for area and perimeter of rectangles. Lesson 2: Solve multiplicative comparison word problems by applying the area and perimeter formulas. Lesson 3: Demonstrate understanding of area and perimeter formulas by solving multi-step real-world problems.	3
4.4C **4.4D** 4.2A 4.4B 4.4H 4.5A	1.C 2.E 2.G 3.E 3.F 4.C 5.B	B	**Multiplication by 10, 100, and 1,000** Lesson 4: Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically. Lesson 5: Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns. Lesson 6: Multiply two-digit multiples of 10 by two-digit multiples of 10 with the area model.	3
4.4C **4.4D** 4.2A 4.4B 4.4H 4.5A	1.C 1.E 2.C 2.D 3.E 3.F 4.B 4.G 5.B	C	**Multiplication of up to Four Digits by Single-Digit Numbers** Lesson 7: Use place value disks to represent two-digit by one-digit multiplication. Lesson 8: Extend the use of place value disks to represent three- and four-digit by one-digit multiplication. Lessons 9–10: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm. Lesson 11: Connect the area model and the partial products method to the standard algorithm.	5
4.4C **4.4D** **4.4H** **4.5A** 4.4G	1.C 1.H 2.I 3.F 4.G 5.A 5.B 5.D 5.E	D	**Multiplication Word Problems** Lesson 12: Solve two-step word problems, including multiplicative comparison. Lesson 13: Use multiplication, addition, or subtraction to solve multi-step word problems.	2

TEKS	ELPS	Topics and Objectives		Days
			Mid-Module Assessment: Topics A–D (review 1 day, assessment ½ day, return ½ day)	2
4.4E **4.4F** 4.4G 4.4H 4.5A	1.C 1.D 1.F 2.E 2.I 3.F 3.H 4.G 5.B	E	**Division of Tens and Ones with Successive Remainders** Lesson 14: Solve division word problems with remainders. Lesson 15: Understand and solve division problems with a remainder using the array and area models. Lesson 16: Understand and solve two-digit dividend division problems with a remainder in the ones place by using place value disks. Lesson 17: Represent and solve division problems requiring decomposing a remainder in the tens. Lesson 18: Find whole number quotients and remainders. Lesson 19: Explain remainders by using place value understanding and models. Lesson 20: Solve division problems without remainders using the area model. Lesson 21: Solve division problems with remainders using the area model.	8
4.4E **4.4F** **4.4H** **4.5A** 4.2A 4.4B 4.4G	1.B 1.C 1.H 2.D 2.E 3.E 3.F 3.I 4.G 5.B 5.F	F	**Division of Thousands, Hundreds, Tens, and Ones** Lesson 22: Divide multiples of 10, 100, and 1,000 by single-digit numbers. Lesson 23: Represent and solve division problems with up to a three-digit dividend numerically and with place value disks requiring decomposing a remainder in the hundreds place. Lesson 24: Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically.	8

TEKS	ELPS	Topics and Objectives		Days
			Lesson 25: Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times. Lesson 26: Solve division problems with a zero in the dividend or with a zero in the quotient. Lesson 27: Interpret division word problems as either number of groups unknown or group size unknown. Lesson 28: Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9. Lesson 29: Explain the connection of the area model of division to the long division algorithm for three- and four-digit dividends.	
4.4C **4.4D** 4.4G 4.4H 4.5A 4.5C 4.5D	1.C 1.H 2.E 2.I 3.D 3.E 3.H 4.C 4.G	G	**Multiplication of Two-Digit by Two-Digit Numbers** Lesson 30: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart. Lesson 31: Multiply two-digit multiples of 10 by two-digit numbers using the area model. Lesson 32: Multiply two-digit by two-digit numbers using four partial products. Lessons 33–34: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.	5
			End-of-Module Assessment: Topics A–G (review 1 day, assessment ½ day, return ½ day, remediation or further applications 1 day)	3
Total Number of Instructional Days				**39**

Terminology

New or Recently Introduced Terms

- Associative property (e.g., 96 = 3 × (4 × 8) = (3 × 4) × 8)
- Distributive property (e.g., 64 × 27 = (60 × 20) + (60 × 7) + (4 × 20) + (4 × 7))
- Divisor (the number by which another number is divided)
- Formula (a mathematical rule expressed as an equation with numbers and/or variables)
- Long division (process of dividing a large dividend using several recorded steps)

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GRADE 4 • MODULE 4

Angle Measure and Plane Figures

Grade 4 • Module 4

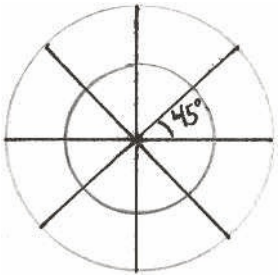
Angle Measure and Plane Figures

OVERVIEW

This 21-day module introduces points, lines, line segments, rays, and angles, as well as the relationships between them. Students construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. With angle measure playing a key role in the work throughout the module, students learn how to create and measure angles, as well as how to create and solve equations to find unknown angle measures. In these problems, where the unknown angle is represented by a letter, students explore both measuring the unknown angle with a protractor and reasoning through the solving of an equation. This connection between the measurement tool and the numerical work lays an important foundation for success with middle-school geometry and algebra. Through decomposition and composition activities, as well as an exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures.

Topic A begins with students drawing points, lines, line segments, and rays, as well as identifying these in various contexts and within familiar figures. Students recognize that two rays sharing a common endpoint form an angle (**4.6A**). They create right angles through a paper-folding activity, identify right angles in their environment, and see that one angle can be greater (obtuse) or less (acute) than a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines as they define, draw, and recognize intersecting, perpendicular, and parallel lines (**4.6A**).

In Topic B, students explore the definition of degree measure, beginning with a circular protractor. By dividing the circumference of a circle into 360 equal parts, they recognize one part as representing 1 degree (**4.7A, 4.7B**). Through exploration, students realize that, although the size of a circle may change, an angle spans an arc, representing a constant fraction of the circumference. By carefully distinguishing the attribute of degree measure from that of length measure, the common misconception that degrees are a measure of length is avoided. Armed with their understanding of the degree as a unit of measure, students use various types of protractors to measure angles to the nearest degree and to sketch angles of a given measure (**4.7C, 4.7D**). The idea that an angle measures the amount of *turning* in a particular direction and the notion that the measure of an angle represents a fraction of a circle that is *cut out* are explored as students recognize familiar angles in varied contexts (**4.6A, 4.7A, 4.7B**). Because the TEKS standard governing the illustration of the measure of an angle (**4.7A**) is not a tested standard, both understandings of angle measure are included in this module.



Topic C begins by decomposing 360° using pattern blocks, allowing students to see that a group of angles meeting at a point with no spaces or overlaps add up to 360°. With this new understanding, students now discover that the combined measure of two adjacent angles on a line is 180°, that the combined measure of two adjacent angles meeting to form a right angle is 90°, and that vertically opposite angles have the same measure. These properties are then used to solve unknown angle problems (**4.7E**). The terms "supplementary angle" and "complementary angle" are included in order to expose students to this vocabulary but are not assessed.

An introduction to symmetry opens Topic D as students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry (**4.6B**). Given one half of a line-symmetric figure and the line of symmetry, students draw the other half of the figure. This leads to their work with triangles. Students are introduced to the precise definition of a triangle and then classify triangles based on angle measure (**4.6C**). Students construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles). Finally, students explore the definitions of familiar quadrilaterals and classify them based on their attributes, including angle measure and parallel and perpendicular lines (**4.6D**). This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. The topic concludes as students compare and analyze two-dimensional figures according to their properties and use grid paper to construct two-dimensional figures given a set of criteria.

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Notes on Pacing for Differentiation

Module 4 may be taught after Module 6 and lessons truncated using the Preparing a Lesson protocol (see Module 1 Overview). This would change the order of the modules to the following: Modules 1, 2, 3, 5, 6, 4, and 7.

Alternately, Module 4's lessons may be truncated by teaching Topic A simultaneously with Module 3 during an art class.

Topics B and C could be taught directly following Module 3, prior to Module 5, since they offer excellent scaffolding for the fraction work of Module 5. Topic D could be taught simultaneously with Module 5, 6, or 7 during an art class when students are served well with hands-on, rigorous experiences.

Keep in mind that Topics B and C of this module are foundational to Grade 7's missing angle problems.

Focus Grade Level Standards

Geometry and Measurement

The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

4.6A identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;

4.6B identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;

4.6C apply knowledge of right angles to identify acute, right, and obtuse triangles;

4.6D classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Geometry and Measurement

The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

4.7A illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;

4.7B illustrate degrees as the units used to measure an angle, where 1/360 of any circle is one degree and an angle that "cuts" n/360 out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;

4.7C determine the approximate measures of angles in degrees to the nearest whole number using a protractor;

4.7D draw an angle with a given measure;

4.7E determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.

Foundational Standards

The student is expected to:

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts;

3.5A represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;

3.5B represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

MPS(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

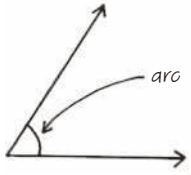
TEKS	ELPS	Topics and Objectives		Days
4.6A 4.6C	1.C 1.F 2.C 2.E 2.I 3.D 3.F 4.F 4.H 5.G	A	**Lines and Angles** Lesson 1: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures. Lesson 2: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles. Lesson 3: Identify, define, and draw perpendicular lines. Lesson 4: Identify, define, and draw parallel lines.	4
4.7A **4.7B** **4.7C** **4.7D**	1.C 1.E 2.C 2.E 2.G 2.I 3.E 4.F 5.G	B	**Angle Measurement** Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor. Lesson 6: Use varied protractors to distinguish angle measure from length measurement. Lesson 7: Measure and draw angles. Sketch given angle measures, and verify with a protractor. Lesson 8: Identify and measure angles as turns and recognize them in various contexts.	4
			Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further application 1 day)	2
4.7E	1.C 2.E 2.H 2.I 3.E 3.H 4.D	C	**Problem Solving with the Addition of Angle Measures** Lesson 9: Decompose angles using pattern blocks. Lessons 10–11: Use the addition of adjacent angle measures to solve problems using a letter for the unknown angle measure.	3

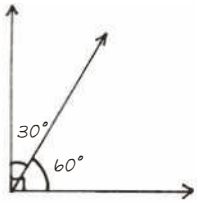
TEKS	ELPS	Topics and Objectives		Days
4.6A **4.6B** **4.6C** **4.6D**	1.C 1.E 1.H 2.C 2.E 2.F 2.I 3.E 4.F 5.G	D	**Two-Dimensional Figures and Symmetry** Lesson 12: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry. Lesson 13: Analyze and classify triangles based on angle measure. Lesson 14: Define and construct triangles from given criteria. Lesson 15: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size. Lesson 16: Reason about attributes to construct quadrilaterals on square or triangular grid paper.	5
			End-of-Module Assessment: Topics A–D (assessment 1 day, return 1 day, remediation or further application 1 day)	3
Total Number of Instructional Days				**21**

Terminology

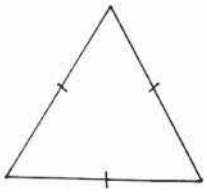
New or Recently Introduced Terms

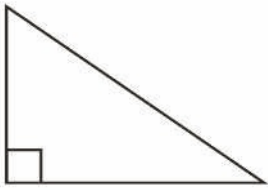
- Acute angle (angle with a measure of less than 90°)
- Acute triangle (triangle with all interior angles measuring less than 90°)
- Adjacent angle (Two angles $\angle AOC$ and $\angle COB$, with a common side $\overrightarrow{OC}$, are *adjacent angles* if C is in the interior of $\angle AOB$.)
- Angle (union of two different rays sharing a common vertex, e.g., $\angle ABC$)
- Arc (connected portion of a circle)





complementary angles
30° + 60° = 90°


acute triangle


right triangle

- Collinear (Three or more points are *collinear* if there is a line containing all of the points; otherwise, the points are *non-collinear*.)

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GRADE 4 • MODULE 5

Fraction Equivalence, Ordering, and Operations

Grade 4 • Module 5

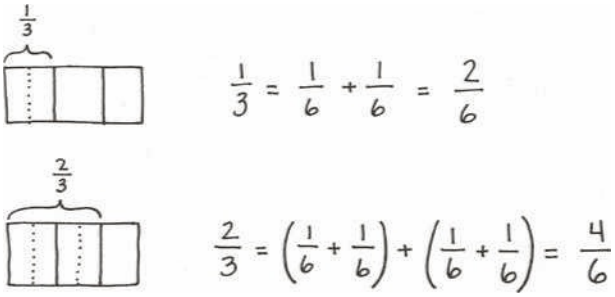
Fraction Equivalence, Ordering, and Operations

OVERVIEW

In this 37-day module, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations.

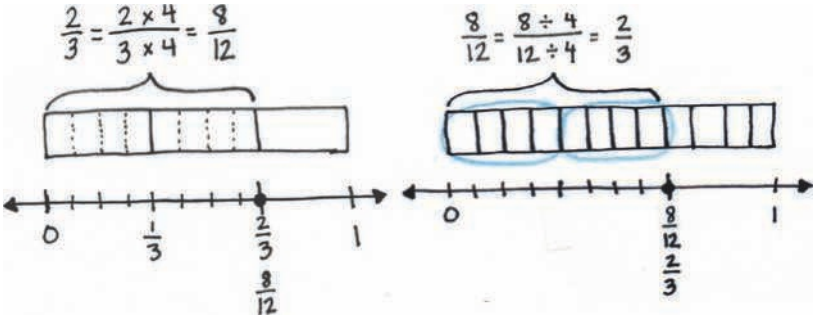
Students begin Topic A by decomposing fractions and creating strip diagrams to represent them as sums of fractions with the same denominator in different ways (e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} + \frac{2}{5}$) (**4.3A, 4.3B**). For example, just as 3 twos = 2 + 2 + 2, so does 3 fourths = $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

As students continue working with decomposition, they represent familiar unit fractions as the sum of smaller unit fractions. A folded paper activity allows them to see that, when the number of fractional parts in a whole increases, the size of the parts decreases. They proceed to investigate this concept with the use of strip diagrams and area models. Reasoning enables them to explain why two different fractions can represent the same portion of a whole (**4.3C**).

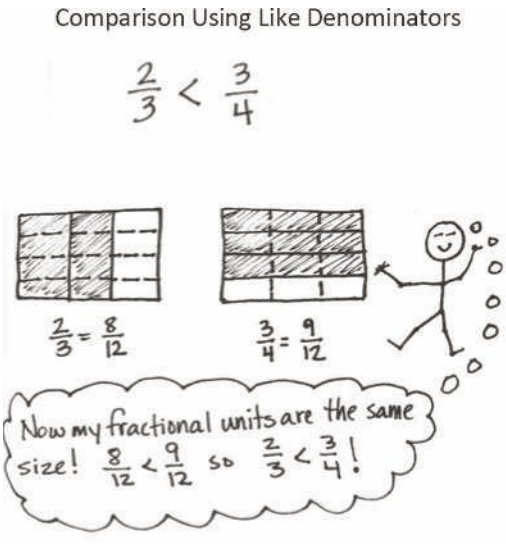


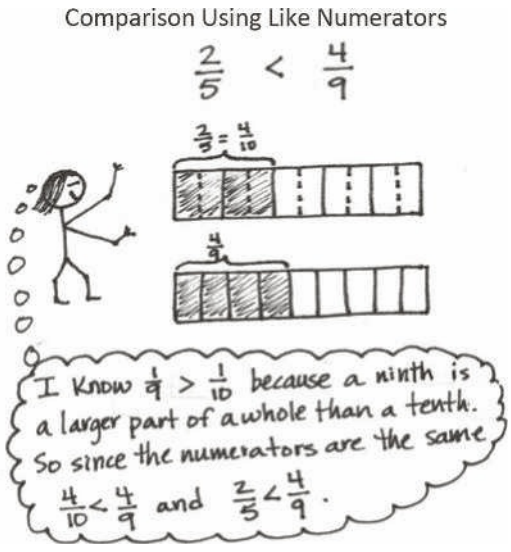
In Topic B, students use strip diagrams and area models to analyze their work from earlier in the module and begin using multiplication to create an equivalent fraction that comprises smaller units, e.g., $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ (**4.3C**). Based on the use of multiplication, they reason that division can be used to create a fraction that comprises larger units (or a single unit) equivalent to a given fraction (e.g., $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$). Their work is justified using area models and strip diagrams and, conversely, multiplication is used to test for and/or verify equivalence. Students use the strip diagram to transition to modeling equivalence on the number line.

They see that, by multiplying, any unit fraction length can be partitioned into *n* equal lengths and that doing so multiplies both the total number of fractional units (the denominator) and number of selected units (the numerator) by *n*. They also see that there are times when fractional units can be grouped together, or divided, into larger fractional units. When that occurs, both the total number of fractional units and number of selected units are divided by the same number.



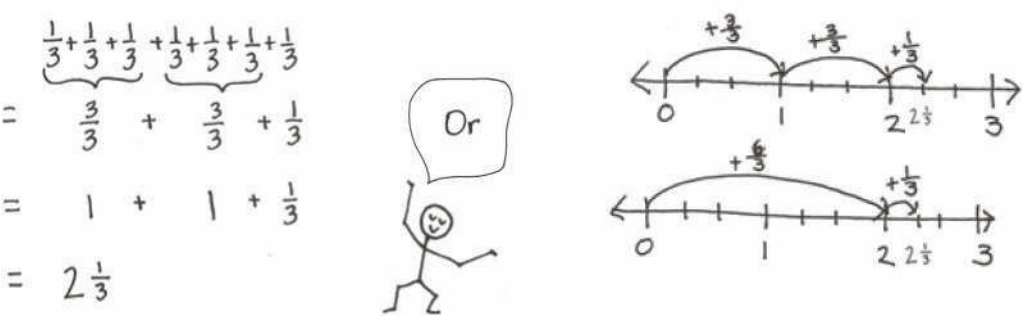
In Grade 3, students compared fractions using fraction strips and number lines with the same denominators. In Topic C, they expand on comparing fractions by reasoning about fractions with unlike denominators. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, $\frac{1}{2}$, or 1) on the number line. Alternatively, students compare using the same numerators. They find that the fraction with the greater denominator is the lesser fraction since the size of the fractional unit is smaller as the whole is decomposed into more equal parts (e.g., $\frac{1}{5} > \frac{1}{10}$; therefore $\frac{3}{5} > \frac{3}{10}$). Throughout the process, their reasoning is supported using strip diagrams and number lines in cases where one numerator or denominator is a factor of the other, such as $\frac{1}{5}$ and $\frac{1}{10}$ or $\frac{2}{3}$ and $\frac{5}{6}$. When the units are unrelated, students use area models and multiplication, the general method pictured below to the left, whereby two fractions are expressed in terms of the same denominators. Students also reason that comparing fractions can only be done when referring to the same whole, and they record their comparisons using the comparison symbols <, >, and = (**4.3D**).





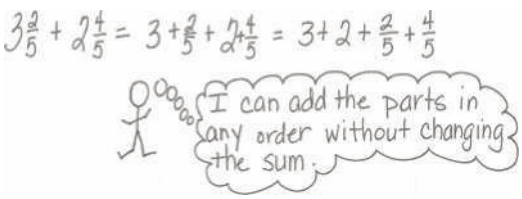
In Topic D, students apply their understanding of whole number addition (the combining of like units) and subtraction (finding an unknown part) to work with fractions (**4.3A, 4.3F**). They see through visual models that, if the units are the same, computation can be performed immediately, e.g., 2 bananas + 3 bananas = 5 bananas and 2 eighths + 3 eighths = 5 eighths. They see that, when subtracting fractions from one whole, the whole is decomposed into the same units as the part being subtracted, e.g., $1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$. Students practice adding more than two fractions and model fractions in word problems using strip diagrams (**4.3E, 4.3G**).

At the beginning of Topic E, students use decomposition and visual models to add and subtract fractions less than 1 to or from whole numbers, e.g., $4 + \frac{3}{4} = 4\frac{3}{4}$ and $4 - \frac{3}{4} = (3 + 1) - \frac{3}{4}$. They use addition and multiplication to build fractions greater than 1 and represent them on the number line.

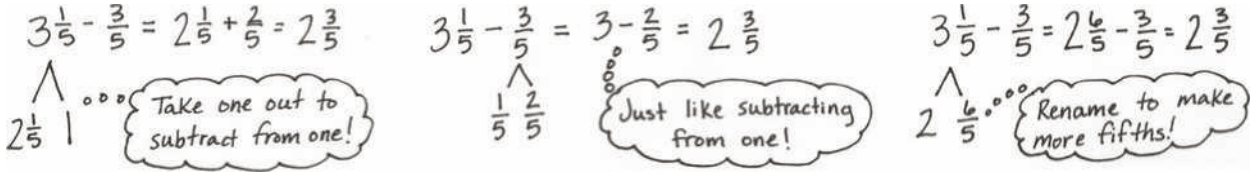


Students then use these visual models and decompositions to reason about the various forms in which a fraction greater than or equal to 1 may be presented, both as fractions and mixed numbers. They practice converting between these forms and begin understanding the usefulness of each form in different situations. Through this understanding, the common misconception that every improper fraction must be converted to a mixed number is avoided. Next, students compare fractions greater than 1, building on their rounding skills and using understanding of benchmarks to reason about which of two fractions is greater (**4.3D**). This activity continues to build understanding of the relationship between the numerator and denominator of a fraction. Students progress to finding and using like denominators or numerators to compare and order mixed numbers. They apply their skills of comparing numbers greater than 1 by solving word problems (**4.3E, 4.3G**) requiring the interpretation and analysis of data presented in dot plots (**4.9A, 4.9B**). Students use addition and subtraction strategies to solve the problems, as well as decomposition and modeling to compare numbers in the data sets.

In Topic F, students estimate sums and differences of mixed numbers, rounding before performing the actual operation to determine what a reasonable outcome is. They proceed to use decomposition to add and subtract mixed numbers (**4.3E**). This work builds on their understanding of a mixed number being the sum of a whole number and fraction.



Using unit form, students add and subtract like units first (e.g., ones and ones, fourths and fourths). Students use decomposition, shown with number bonds, in mixed number addition to make one from fractional units before finding the sum. When subtracting, students learn to decompose the minuend or subtrahend when there are not enough fractional units from which to subtract. Alternatively, students can rename the subtrahend, giving more units to the fractional units, which connects to whole number subtraction when renaming 9 tens 2 ones as 8 tens 12 ones.



Topic F closes with solving multiplicative comparison word problems involving fractions (**4.3A, 4.3B**).

Although students use repeated addition to solve these multiplicative comparison problems, they are able to make connections to the structure of other multiplicative comparison situations encountered in Grade 3 and earlier in Grade 4.

The Mid-Module Assessment follows Topic D and the End-of-Module Assessment follows Topic F.

Notes on Pacing for Differentiation

For Module 5, consider the following modifications and omissions. Study the objectives and the sequence of problems within Lessons 1 and 2, and then consolidate them. Omit Lesson 3. Instead, in Lesson 5, embed the contrast of the decomposition of a fraction using the strip diagram versus using the area model. Note that the area model's cross hatches are used to transition to multiplying to generate equivalent fractions. This model is also used in Module 6 to add decimals. The use of this model continues extensively in Grade 5. In Grade 5, Module 3 it's used to add related fractions and then to add and subtract fractions with unlike denominators; and used again to multiply a fraction by a fraction in Grade 6.

Lesson 31 offers an extension of fraction addition. This lesson applies fraction addition to multiplicative comparison story problems. If time is short, this lesson might be omitted.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:

4.3A represent a fraction $\frac{a}{b}$ as a sum of fractions $\frac{1}{b}$, where a and b are whole numbers and $b > 0$, including when $a > b$;

4.3B decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations;

4.3C determine if two given fractions are equivalent using a variety of methods;

4.3D compare two fractions with different numerators and different denominators and represent the comparison using the symbols >, =, or <;

4.3E represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations;

4.3F evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1, referring to the same whole;

4.3G represent fractions and decimals to the tenths or hundredths as distances from zero on a number line.

Data Analysis

The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

4.9A represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions;

4.9B solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.

Foundational Standards

The student is expected to:

3.3A represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines;

3.3B determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line;

3.3C explain that the unit fraction $\frac{1}{b}$ represents the quantity formed by one part of a whole that has been partitioned into b equal parts where b is a non-zero whole number;

3.3D compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts $\frac{1}{b}$;

3.3E solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8;

3.3F represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;

3.3G explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model;

3.3H compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models;

3.6E decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape;

3.7A represent fractions of halves, fourths, and eighths as distances from zero on a number line.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
4.3A **4.3B** **4.3C** 4.3F	1.C 2.C 2.E 2.I 3.A 3.E 3.H 4.G 5.C	A	**Decomposition and Fraction Equivalence** Lessons 1–2: Decompose fractions as a sum of unit fractions using strip diagrams. Lesson 3: Decompose fractions into sums of smaller unit fractions using strip diagrams. Lesson 4: Decompose unit fractions using area models to show equivalence. Lesson 5: Decompose fractions using area models to show equivalence.	5
4.3C 4.3B 4.3G	1.C 1.H 2.C 2.E 2.I 3.E 3.H 4.G 5.F	B	**Fraction Equivalence Using Multiplication and Division** Lessons 6–7: Use the area model and multiplication to show the equivalence of two fractions. Lessons 8–9: Use the area model and division to show the equivalence of two fractions. Lesson 10: Explain fraction equivalence using a strip diagram and the number line, and relate that to the use of multiplication and division.	5
4.3D	1.C 1.E 2.E 2.I 3.E 3.H 4.B 5.G	C	**Fraction Comparison** Lessons 11–12: Reason using benchmarks to compare two fractions on the number line. Lessons 13–14: Find common units or number of units to compare two fractions.	4
4.3A **4.3B** **4.3E** **4.3F** **4.3G** 4.3C	1.C 1.G 2.E 2.I 3.A 3.E 3.G 3.H 4.B 5.F	D	**Fraction Addition and Subtraction** Lesson 15: Use visual models to add and subtract two fractions with the same units. Lesson 16: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole. Lesson 17: Add and subtract more than two fractions. Lesson 18: Solve word problems involving addition and subtraction of fractions.	4

TEKS	ELPS		Topics and Objectives	Days
			Mid-Module Assessment: Topics A–D (assessment 1 day, return 1 day, remediation or further applications 1 day)	3
4.3A **4.3B** **4.3D** **4.3E** **4.3F** **4.3G** **4.9A** **4.9B** 4.3C	1.C 1.H 2.C 2.I 3.E 3.G 3.H 4.C 4.K 5.F	E	**Extending Fraction Equivalence to Fractions Greater Than 1** Lesson 19: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models. Lessons 20–21: Decompose and compose fractions greater than 1 to express them in various forms. Lesson 22: Compare fractions greater than 1 by reasoning using benchmark fractions. Lesson 23: Compare fractions greater than 1 by creating common numerators or denominators. Lesson 24: Solve word problems with dot plots.	6
4.3E **4.3F**	1.C 1.E 1.H 2.E 2.H 2.I 3.H 4.C 5.G	F	**Addition and Subtraction of Fractions by Decomposition** Lesson 25: Estimate sums and differences using benchmark numbers. Lesson 26: Add a mixed number and a fraction. Lesson 27: Add mixed numbers. Lesson 28: Subtract a fraction from a mixed number. Lesson 29: Subtract a mixed number from a mixed number. Lesson 30: Subtract mixed numbers. Lesson 31: Solve multiplicative comparison word problems involving fractions.	7
			End-of-Module Assessment: Topics A–F (assessment 1 day, return 1 day, remediation or further applications 1 day)	3
Total Number of Instructional Days				**37**

Table of Contents

GRADE 4 • MODULE 6

Decimal Fractions and Financial Literacy

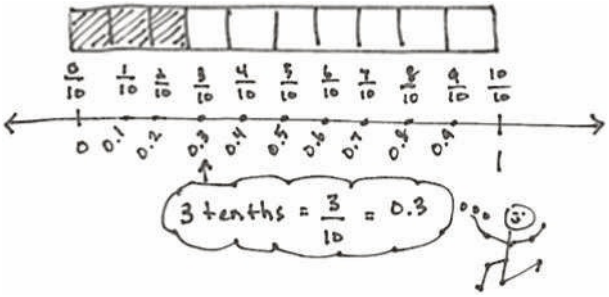
Grade 4 • Module 6

Decimal Fractions and Financial Literacy

OVERVIEW

This 26-day module gives students their first opportunity to explore decimal numbers via their relationship to decimal fractions, expressing a given quantity in both fraction and decimal forms. Utilizing the understanding of fractions developed throughout Module 5, students apply the same reasoning to decimal numbers, building a solid foundation for Grade 5 work with decimal operations. Previously referred to as whole numbers, all numbers written in the base-ten number system with place value units that are powers of 10 are henceforth referred to as decimal numbers, a set which now includes tenths and hundredths (e.g., 1, 15, 248, 0.3, 3.02, and 24.35). Throughout this module, students explore the relationship between the powers of 10 shown on the place value chart. They express these relationships in terms of ten times as much or 1 tenth as much (**4.2A**).

In Topic A, students use their understanding of fractions to explore tenths. At the opening of the topic, they use metric measurement to see tenths in relation to different whole units: centimeters, meters, kilograms, and liters. Students explore, creating and identifying tenths of various wholes, as they draw lines of specified length, identify the weight of objects, and read the level of liquid measurements. Students connect these concrete experiences pictorially as tenths are represented on the number line and with strip diagrams as pictured to the right. Students express tenths as decimal fractions and are introduced to decimal notation. They write statements of equivalence in unit, fraction, and decimal forms (e.g., 3 tenths = $\frac{3}{10}$ = 0.3). Next, students return to the use of metric measurement to investigate decimal fractions greater than 1. Using a centimeter ruler, they draw lines that measure, for example, $2\frac{4}{10}$ or $6\frac{8}{10}$ centimeters. Using the area model, students see that numbers containing a whole number and fractional part (i.e., mixed numbers) can also be expressed using decimal notation, provided that the fractional part can be converted to a decimal number. Students use place value disks to represent the value of each digit in a decimal number. Just as they wrote whole numbers in expanded notation using multiplication, students write the value of a decimal number in expanded notation using fractions and decimals; for example, 2 ones 4 tenths = $2\frac{4}{10} = (2 \times 1) + (4 \times \frac{1}{10})$ and 2.4 = (2 × 1) + (4 × 0.1). Additionally, students plot decimal numbers on the number line (**4.2E, 4.2G, 4.2H**).

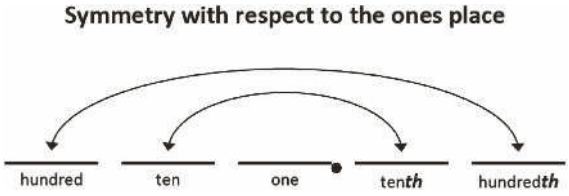


Students decompose tenths into 10 equal parts to create hundredths in Topic B. Through the decomposition of a meter, students identify 1 centimeter as 1 hundredth of a meter. As students count up by hundredths, they realize the equivalence of 10 hundredths and 1 tenth and go on to represent them as both decimal

fractions and as decimal numbers (**4.3C**). Students use area models, strip diagrams, and place value disks on a place value chart to see and model the equivalence of numbers involving units of tenths and hundredths. They express the value of the number in both decimal and fraction expanded notations.

$$31\ \tfrac{46}{100} = (3 \times 10) + (1 \times 1) + (4 \times \tfrac{1}{10}) + (6 \times \tfrac{1}{100})$$
$$31.46 = (3 \times 10) + (1 \times 1) + (4 \times 0.1) + (6 \times 0.01)$$

Close work with the place value chart helps students see that place value units are not symmetric about the decimal point—a common misconception that often leads students to mistakenly believe there is a *oneths* place. They explore the placement of decimal numbers to hundredths and recognize that the place value chart is symmetric about the ones column. This understanding helps students recognize that, even as we move to the units on the right side of the decimal on the place value chart, a column continues to represent a unit 10 times as large as that of the column to its right. This understanding builds on the place value work done in Module 1 and enables students to understand that 3.2, for example, might be modeled as 3 ones 2 tenths, 32 tenths, or 320 hundredths. Topic B concludes with students using their knowledge of fraction equivalence to work with decimal numbers expressed in unit form, fraction form, and decimal form (**4.2G**).

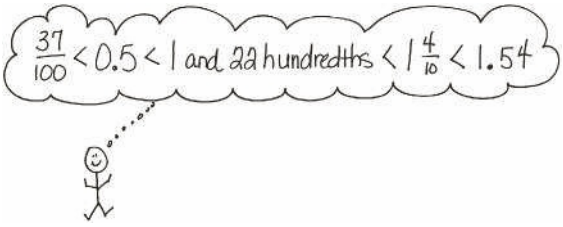


The focus of Topic C is comparison of decimal numbers (**4.2F**). To begin, students work with concrete representations of measurements. They see measurement of length on meter sticks, of mass using a scale, and of volume using graduated cylinders. In each case, students record the measurements on a place value chart and then compare them. They use their understanding of metric measurement and decimals to answer questions, such as, "Which is greater? Less? Which is longer? Shorter? Which is heavier? Lighter?" Comparing the decimals in the context of measurement supports students' justification of their comparisons and grounds their reasoning, while at the same time setting them up for work with decimal comparison at a more concrete level. Next, students use area models and number lines to compare decimal numbers and use the <, >, and = symbols to record their comparisons. All of their work with comparisons at the pictorial level helps to eradicate the common misconception that is often made when students assume a greater number of hundredths must be greater than a lesser number of tenths. For example, when comparing 7 tenths and 27 hundredths, students recognize that 7 tenths is greater than 27 hundredths because, as in any comparison, one must consider the *size of the units*. Students go on to arrange mixed groups of decimal fractions in unit, fraction, and decimal forms in order from greatest to least or least to greatest. They use their understanding of different ways of expressing equivalent values to arrange a set of decimal fractions as pictured below.

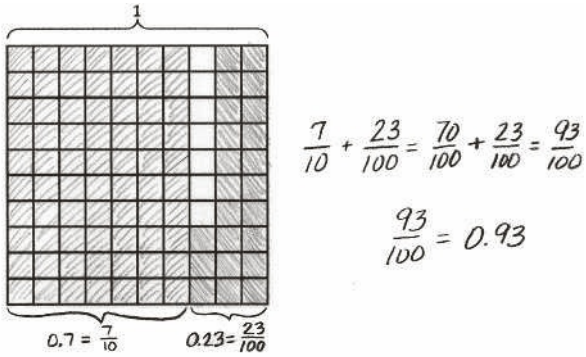
Mass of Rice Bags (kilograms)

Rice Bag	ones	.	tenths	hundredths
A	0	.	1	0
B	0	.	6	5
C	0	.	7	
D	0	.	4	6

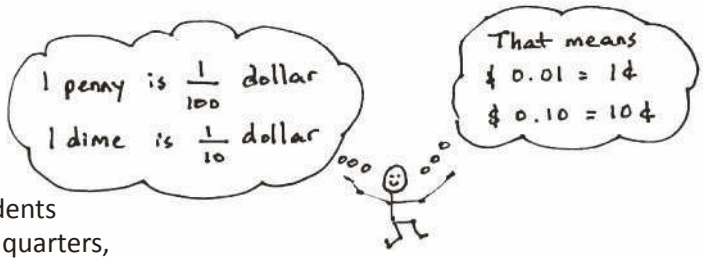
0.7 kg, 0.65 kg, 0.46 kg, 0.1 kg

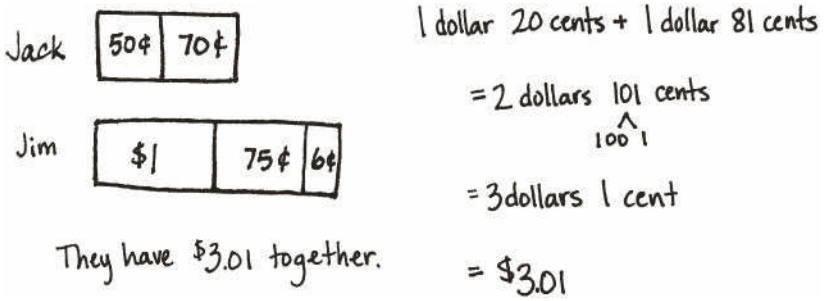


Topic D introduces the addition and subtraction of decimals (**4.4A**). Students add and subtract tenths and hundredths by converting fraction notation to equivalent decimal notation (**4.2G**). Place value understanding, and the standard algorithm are employed to find sums and differences. Students use the same process to add and subtract mixed numbers involving decimal units. They then apply their new knowledge to solve word problems involving metric measurements (**4.8C**).



Students conclude their work with decimal fractions in Topic E by applying their knowledge to the real-world context of money. They recognize 1 penny as $\frac{1}{100}$ dollar, 1 dime as $\frac{1}{10}$ dollar, and 1 quarter as $\frac{25}{100}$ dollar. They apply their understanding of tenths and hundredths to write given amounts of money in both fraction and decimal forms. To do this, students decompose a given amount of money into dollars, quarters, dimes, and pennies and express the amount as a decimal fraction and decimal number. Students then add various numbers of coins and dollars using Grade 2 knowledge of the equivalence of 100 cents to 1 dollar. Addition and subtraction word problems are solved using unit form, adding dollars and cents. Multiplication and division word problems are solved using cents as the unit (**4.8C**). The final answer in each word problem is converted from cents into a decimal using a dollar symbol for the unit. For example, *Jack has 2 quarters and 7 dimes. Jim has 1 dollar, 3 quarters, and 6 pennies. How much money do they have together? Write your answer as a decimal.*





After students have studied decimal notation and money, there is a natural segue to studying personal financial literacy. Students investigate different types of financial institutions, and the advantages and disadvantages of options available for saving money. They explore fixed and variable expenses and how this relates to allowance. Finally, students solve word problems that include calculating profits (**4.10A-E**).

Notes on Pacing for Differentiation

In Module 6, students explore decimal numbers for the first time by means of the decimal numbers' relationship to decimal fractions. Module 6 builds directly from Module 5 and is foundational to students' Grade 5 work with decimal operations. Therefore, it is not recommended to omit any lessons from Module 6.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:

4.2A interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;

4.2B represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals;

4.2E represent decimals, including tenths and hundredths, using concrete and visual models and money;

4.2F compare and order decimals using concrete and visual models to the hundredths;

4.2G relate decimals to fractions that name tenths and hundredths;

4.2H determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line.

Number and Operations

The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:

4.3C determine if two given fractions are equivalent using a variety of methods.

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:

4.4A add and subtract whole numbers and decimals to the hundredths place using the standard algorithm.

Geometry and Measurement

The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:

4.8C solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.

Personal Financial Literacy

The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

4.10A distinguish between fixed and variable expenses;

4.10B calculate profit in a given situation;

4.10C compare the advantages and disadvantages of various savings options;

4.10D describe how to allocate a weekly allowance among spending; saving, including for college; and sharing;

4.10E describe the basic purpose of financial institutions, including keeping money safe, borrowing money, and lending.

Foundational Standards

The student is expected to:

2.5A determine the value of a collection of coins up to one dollar;

2.5B use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins;

3.3A represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines;

3.3B determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line;

3.3C explain that the unit fraction $\frac{1}{b}$ represents the quantity formed by one part of a whole that has been partitioned into b equal parts where b is a non-zero whole number;

3.3F represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;

3.3G explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model;

3.3H compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models;

3.4F recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;

3.4G	use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;
3.7A	represent fractions of halves, fourths, and eighths as distances from zero on a number line;
3.7D	determine when it is appropriate to use measurements of liquid volume (capacity) or weight;
3.7E	determine liquid volume (capacity) or weight using appropriate units and tools;
3.9C	identify the costs and benefits of planned and unplanned spending decisions;
3.9E	list reasons to save and explain the benefit of a savings plan, including for college.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(D)	communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
MPS(E)	create and use representations to organize, record, and communicate mathematical ideas;
MPS(F)	analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS		Topics and Objectives	Days
4.2A **4.2E** **4.2G** **4.2H** 4.4B 4.8A 4.8B	1.C 2.A 2.E 2.I 3.A 3.E 3.H 4.B	A	**Exploration of Tenths** Lesson 1: Use metric measurement to model the decomposition of one whole into tenths. Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers. Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded notation.	3
4.2B **4.2E** **4.2G** **4.2H** **4.3C** 4.2A 4.2F 4.8B	1.C 2.A 2.C 2.E 2.I 3.A 3.E 3.H 4.I	B	**Tenths and Hundredths** Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths. Lesson 5: Model the equivalence of tenths and hundredths using the area model and place value disks. Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms. Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded notation and on the place value chart. Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.	5
			Mid-Module Assessment: Topics A–B (assessment 1 day, return 1 day, remediation or further applications 2 days)	4
4.2F 4.8C	1.E 2.E 2.I 3.E 3.J 4.D 5.B	C	**Decimal Comparison** Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions. Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =. Lesson 11: Compare and order mixed numbers in various forms.	3
4.2G **4.3C** **4.4A** 4.2E 4.2H 4.3E 4.8B 4.8C	1.C 1.F 2.E 2.G 2.I 4.A 4.G	D	**Addition with Tenths and Hundredths** Lesson 12: Apply place value understanding to add tenths and hundredths. Lesson 13: Apply place value understanding to subtract tenths and hundredths. Lesson 14: Solve word problems involving the addition and subtraction of measurements in decimal form.	3

TEKS	ELPS	Topics and Objectives		Days
4.8C **4.10A** **4.10B** **4.10C** **4.10D** **4.10E** 4.2E 4.2G 4.2H 4.3C	1.A 1.D 2.E 2.I 3.E 3.H 4.B 5.G	E	**Money Amounts as Decimal Numbers and Financial Literacy** Lesson 15: Express money amounts given in various forms as decimal numbers. Lesson 16: Understand the purpose of financial institutions and advantages and disadvantages of savings options. Lesson 17: Understand the difference between fixed and variable expenses for the purpose of allocating a weekly allowance. Lesson 18: Solve word problems including the calculation of profit.	4
			End-of-Module Assessment: Topics A–E (assessment 1 day, return 1 day, remediation or further applications 2 days)	4
Total Number of Instructional Days				**26**

Terminology

New or Recently Introduced Terms

- Cost (price of goods or services)
- Decimal expanded notation (e.g., (2 × 10) + (4 × 1) + (5 × 0.1) + (9 × 0.01) = 24.59)
- Decimal fraction (a fraction with a denominator of 10, 100, 1,000, etc.)
- Decimal number (a number written using place value units that are powers of 10)
- Decimal point (a period used to separate the whole number part from the fractional part of a decimal number)
- Fixed expense (consistent, set cost)
- Fraction expanded notation (e.g., $(2\times10)+(4\times1)+(5\times\frac{1}{10})+(9\times\frac{1}{100})=24\frac{59}{100}$)
- Hundredth (a place value unit such that 100 hundredths equals 1 one)
- Interest (a sum paid or charged for the use of money)
- Lending/loan (to temporarily give)
- Profit (monetary gain)
- Selling price (the price at which a good or service is sold)
- Spending/spent (to pay or buy)
- Tenth (a place value unit such that 10 tenths equals 1 one)
- Variable expense (cost that can be changed or manipulated frequently)
- Withdrawal (to take money out of a bank account, or the sum of money taken from an account)

Familiar Terms and Symbols[1]

- Expanded form (e.g., 100 + 30 + 5 = 135)
- Fraction (a numerical quantity that is not a whole number, e.g., $\frac{1}{3}$)

[1]These are terms and symbols students have seen previously.

Table of Contents

GRADE 4 • MODULE 7

Exploring Measurement with Multiplication and Data

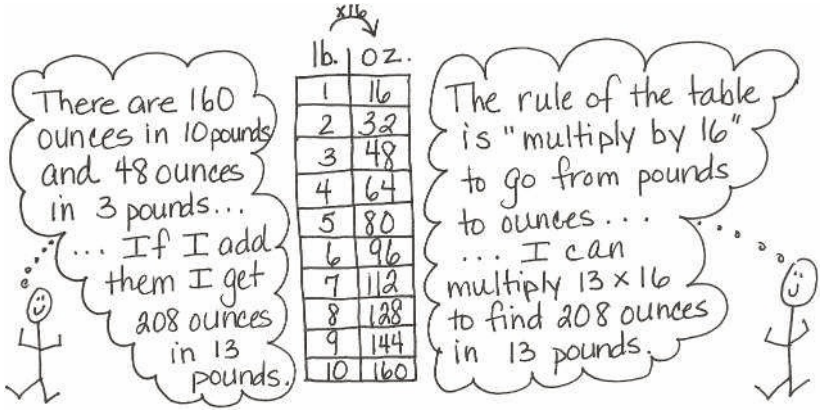
Grade 4 • Module 7

Exploring Measurement with Multiplication and Data

OVERVIEW

In this module, students build their competencies in measurement as they relate multiplication to the conversion of measurement units. Throughout the module, students explore multiple strategies for solving measurement problems involving unit conversion. In every topic in the module, students are given opportunities to engage with input-output tables relating values to their positions in sequences and using given rules to generate number patterns (**4.5B**). This deliberate practice is accomplished through Application Problems and Fluency activities to provide students with multiple exposures to the cognitively demanding task of interpreting relationships presented in tables.

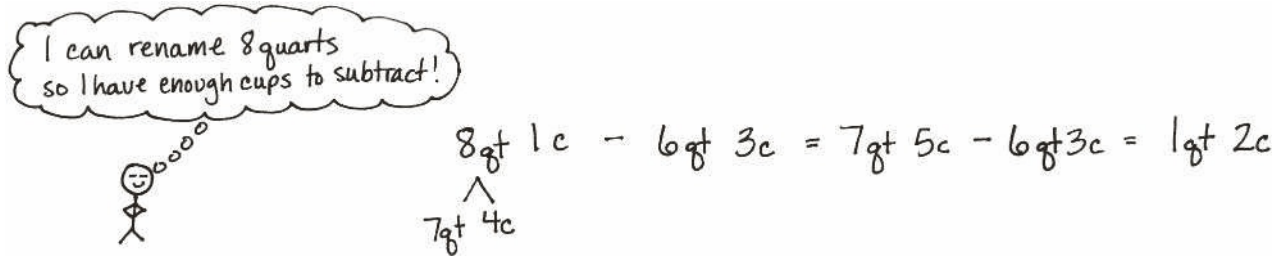
In Topic A, students build on their work in Module 2 with measurement conversions. In Module 2, students converted metric units from large units to small units. In this module, students have opportunities to convert metric units from small units to large units, while also working heavily with customary units. Students use two-column conversion tables (**4.8B**) to practice conversion rates. For example, following a discovery activity where students learn that 16 ounces make 1 pound, students generate a two-column conversion table listing the number of ounces in 1 to 10 pounds. Tables for other measurement units are then generated in a similar fashion. Students then reason about why they do not need to complete the tables beyond 10 of the larger units. They use their multiplication skills from Module 3 to complete the tables and are able to see and explain connections such as (13 × 16) = (10 × 16) + (3 × 16). One student could reason, for example, that, "Since the table shows that there are 160 ounces in 10 pounds and 48 ounces in 3 pounds, I can add them together to tell that there are 208 ounces in 13 pounds." Another student might reason that, "Since there are 16 ounces in each pound, I can use the rule of the table and multiply 13 pounds by 16 to find that there are 208 ounces in 13 pounds."



As the topic progresses, students solve multiplicative comparison word problems. They are then challenged to create and solve their own word problems and to critique the reasoning of their peers (**4.4H, 4.5A**). They share their solution strategies and original problems within small groups, as well as share and critique the problem-solving strategies used by their peers. Through the use of guided questions, students discuss not only how the problems were solved, but also the advantages and disadvantages of using each strategy. They

further discuss what makes one strategy more efficient than another. By the end of Topic A, students have started to internalize the conversion rates through fluency exercises and continued practice.

Topic B builds upon the conversion work from Topic A to add and subtract mixed units of capacity, length, weight, and time. Working with metric and customary units (**4.8A**), students add like units, making comparisons to adding like fractional units, further establishing the importance of deeply understanding the unit. Just as 2 fourths + 3 fourths = 5 fourths, so does 2 quarts + 3 quarts = 5 quarts. 5 fourths can be decomposed into 1 one and 1 fourth, and therefore, 5 quarts can be decomposed into 1 gallon and 1 quart. Students realize the same situation occurs in subtraction. Just as $1-\frac{3}{4}$ must be renamed to $\frac{4}{4}-\frac{3}{4}$ so that the units are alike, students must also rename units of measurements to make like units (1 quart − 3 cups = 4 cups − 3 cups) (**4.8B**). Students go on to add and subtract mixed units of measurements, finding multiple solution strategies, similar to the mixed number work in fractions. With a focus on measurement units of capacity, length, weight, and time, students apply this work to solve multi-step word problems (**4.5A**).



In Topic C, students reason how to convert larger units of measurements with fractional parts into smaller units by using hands-on measurements (**4.8B**). For example, students convert $3\frac{1}{4}$ feet to inches by first finding the number of inches in $\frac{1}{4}$ foot. They partition a length of 1 foot into 4 equal parts and find that $\frac{1}{4}$ foot = 3 inches. They then convert 3 feet to 36 inches and add 3 inches to find that $3\frac{1}{4}$ feet = 39 inches. This work is directly analogous to earlier work with fraction equivalence using the strip diagram, area model, and number line in Topics A, B, and D of Module 5. Students partitioned a whole into 4 equal parts, decomposed 1 part into 3 smaller units, and found 1 fourth to be equal to 3 twelfths. The foot ruler is partitioned with precisely the same reasoning. Students close the topic by using measurements to solve multi-step word problems that require converting larger units into smaller units (**4.5A, 4.8C**).

In Topic D, students collect data with measurements in fractions and represent this data on frequency tables and dot plots (**4.9A**). Next, students represent data on stem-and-leaf plots (**4.9A**). They apply their knowledge of these data representations to solve one- and two-step problems (**4.9B**).

The End-of-Module Assessment follows Topic D.

Students review their year in Topic E by practicing the skills they have learned throughout the modules. Additionally, they create a take-home summer folder. The cover of the folder is transformed into the student's own miniature personal white board, and a collection of activities from the lessons within this topic are placed inside the folder to be practiced throughout the summer. Students practice major skills and concepts learned throughout the year in these final four lessons, including measuring angles and drawing lines, multiplication and division, and addition and subtraction through guided group work, fluency activities, and vocabulary games.

Notes on Pacing for Differentiation

Module 7 affords students the opportunity to use all that they have learned throughout Grade 4 as they first relate multiplication to the conversion of measurement units and then explore multiple strategies for solving measurement problems involving unit conversion. Module 7 ends with practice of the major skills and concepts of the grade as well as the preparation of a take-home summer folder. Therefore, it is not recommended to omit any lessons from Module 7.

Relating values to their positions in sequences through numerical rules and input-output tables (**4.5B**) is covered using the Fluency activities *What's the Rule?* and *Use the Rule*. These Fluency activities are found in Lessons 7, 10, 15, 16, and 17. If modifications are deemed necessary to the listed lessons, be sure to embed these Fluency activities at other points during instruction. Representing problems using input-output tables and numerical expressions to relate a value to its position in a sequence is also found in the Application Problems in Lessons 1, 2, 3, 5, 6, 9, and 14. These problems are important to include, should other modifications be made to these lessons.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:

4.4H solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders.

Algebraic Reasoning

The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

4.5A represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;

4.5B represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule representing the relationship of the values in the resulting sequence and their position in the sequence.

Geometry and Measurement

The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:

4.8A identify relative sizes of measurement units within the customary and metric systems;

4.8B convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table;

4.8C solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.

Data Analysis

The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

4.9A represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions;

4.9B solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.

Foundational Standards

The student is expected to:

3.3F represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;

3.3G explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model;

3.3H compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models;

3.4D determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10;

3.4E represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;

3.4F recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;

3.4G use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

3.4K solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

3.7D determine when it is appropriate to use measurements of liquid volume (capacity) or weight;

3.7E determine liquid volume (capacity) or weight using appropriate units and tools.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

MPS(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

MPS(E) create and use representations to organize, record, and communicate mathematical ideas;

MPS(F) analyze mathematical relationships to connect and communicate mathematical ideas;

MPS(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives		Days
4.4H **4.5A** **4.5B** **4.8A** **4.8B** 4.4C 4.4D 4.8C	1.A 1.C 2.C 2.D 2.E 3.E 3.J 4.I 5.G	A	**Measurement Conversion Tables** Lessons 1–2: Create conversion tables for length, weight, and capacity units using measurement tools, and use the tables to solve problems. Lesson 3: Create conversion tables for units of time, and use the tables to solve problems. Lesson 4: Solve multiplicative comparison word problems using measurement conversion tables. Lesson 5: Share and critique peer strategies.	5
4.4H **4.5A** **4.8A** **4.8B** **4.8C** 4.4C 4.4D 4.4E 4.4F 4.5B	1.A 1.H 2.E 3.E 3.F 3.H 4.I 5.G	B	**Problem Solving with Measurement** Lesson 6: Solve problems involving mixed units of capacity. Lesson 7: Solve problems involving mixed units of length. Lesson 8: Solve problems involving mixed units of weight. Lesson 9: Solve problems involving mixed units of time. Lessons 10–11: Solve multi-step measurement word problems.	6
4.4H **4.5A** **4.8B** **4.8C** 4.4C 4.4D 4.4E 4.4F 4.5B 4.8A	1.C 2.C 2.E 3.E 4.G	C	**Investigation of Measurements Expressed as Mixed Numbers** Lessons 12–13: Use measurement tools to convert mixed number measurements to smaller units. Lesson 14: Solve multi-step word problems involving converting mixed number measurements to a single unit.	3

TEKS	ELPS	Topics and Objectives		Days
4.9A **4.9B** 4.5B	2.C 2.E 3.E 3.H 4.I 5.G	D	**Data Analysis** Lesson 15: Measure and compare pencil lengths to the nearest $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ of an inch, and analyze the data with frequency tables and dot plots. Lesson 16: Compare and analyze data represented in a dot plot and a stem-and-leaf plot. Lesson 17: Problem solving with data and graphs.	3
			End-of-Module Assessment: Topics A–D (assessment 1 day, 1 day return, remediation or further application 2 days)	4
*	1.B 2.E 3.G 4.E 4.I 5.G	E	**Year in Review** Lessons 18–19: Create and determine the area of composite figures. Lesson 20: Practice and solidify Grade 4 fluency. Lesson 21: Practice and solidify Grade 4 vocabulary.	4
Total Number of Instructional Days				**25**

**The Year in Review offers a spiral review of the major work of the current grade level with a view toward success in the succeeding grade level.*

Terminology

New or Recently Introduced Terms

- Cup (c) (customary unit of measure for liquid volume)
- Customary system of measurement (measurement system commonly used in the United States that includes such units as yards, pounds, and gallons)
- Customary unit (e.g., foot, ounce, quart)
- Gallon (gal) (customary unit of measure for liquid volume)
- Key (notation on a data display explaining the value of a unit)
- Metric system of measurement (base-ten system of measurement used internationally that includes such units as meters, kilograms, and liters)
- Metric unit (e.g., kilometer, gram, milliliter)
- Ounce (oz) (customary unit of measure for weight)
- Pint (pt) (customary unit of measure for liquid volume)
- Pound (lb) (customary unit of measure for weight)
- Quart (qt) (customary unit of measure for liquid volume)
- Stem-and-leaf plot (a plot where each data value is split into a leaf and a stem)