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# GRADE 5 • MODULE 1

## Place Value and Decimal Fractions

## Grade 5 • Module 1

# Place Value and Decimal Fractions

## OVERVIEW

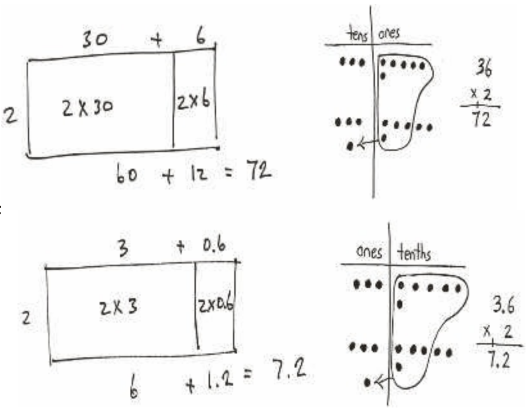
In Module 1, students' understandings of the patterns in the base ten system are extended from Grade 4's work with place value to include decimals to the thousandths place in Topic A (**5.2A**). This work is done through reviewing their knowledge of the relationships between and among adjacent places on the place value chart, e.g., 1 tenth times any digit on the place value chart moves the digit one place value to the right (**4.2A**). Students describe the value of the digits in decimal fractions using these familiar relationships and notice how the value of the digits change as a result of multiplying and dividing by multiples of 10. Toward the module's end, students apply these new understandings as they reason about and perform decimal operations through the hundredths place.

Topic B moves into the naming of decimal fraction numbers in expanded, unit (e.g., 4.23 = 4 ones 2 tenths 3 hundredths), and word forms and concludes with using like units to compare decimal fractions. Students use multiples of 10 and unit fractions to represent expanded notation (e.g., $(2 \times 100) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) = 200.34$) (**5.2A, 5.2B**). Further, students reason about differences in the values of like place value units

and express those comparisons with symbols (>, <, and =). Students generalize their knowledge of rounding whole numbers to round decimal numbers in Topic C, initially using a vertical number line to interpret the result as an approximation and then eventually moving away from the visual model (**5.2C**).

In the latter topics of Module 1, students use the relationships of adjacent units and generalize whole-number algorithms to decimal fraction operations. This work will be continued in Module 2. Topic D gives a brief look back to Grade 4 and uses unit form to connect general methods for addition and subtraction with whole numbers to decimal addition and subtraction (e.g., 7 tens + 8 tens = 15 tens = 150 is analogous to 7 tenths + 8 tenths = 15 tenths = 1.5).

Topic E bridges the gap between Grade 4 work with multiplication and the standard algorithm by focusing on an intermediate step—reasoning about multiplying a decimal by a one-digit whole number. The area model, with which students have had extensive experience since Grade 3, is used as a scaffold for this work.




Topic F concludes Module 1 with a similar exploration of division of decimal numbers by one-digit whole-number divisors. Students solidify their skills with an understanding of the algorithm before moving on to long division involving two-digit divisors in Module 2.

The Mid-Module Assessment follows Topic C. The End-of-Module Assessment follows Topic F.

### Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Consolidate Lessons 9 and 10 because these lessons devote a day each to adding and subtracting with decimals. If students are fluent with addition and subtraction with whole numbers and their understanding of decimal place value is strong (from Grade 4 Module 6 and Grade 5 Module 1 Topic B), practicing both addition and subtraction with decimals can be done in one lesson. Begin assessing students' skill with addition and subtraction with whole numbers during the fluency activity of Lesson 5, and spend a series of days doing so.

## Focus Grade Level Standards

### Number and Operations

**The student applies mathematical process standards to represent, compare, and order positive rational numbers and understand relationships as related to place value. The student is expected to:**

**5.2A** represent the value of the digit in decimals through the thousandths using expanded notation and numerals;

**5.2B** compare and order two decimals to thousandths and represent comparisons using the symbols >, <, or =;

**5.2C** round decimals to tenths or hundredths.

### Number and Operations

**The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy.[1] The student is expected to:**

**5.3A** estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division;

**5.3D** represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;

**5.3E** solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;

**5.3F** represent quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using objects and pictorial models, including area models;

[1]The balance of rational number computation is addressed in other modules.

**5.3G** solve for quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm;

**5.3K** add and subtract positive rational numbers fluently.

### Geometry and Measurement

**The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to:**

**5.7** solve problems by calculating conversions within a measurement system, customary or metric.[2]

## Foundational Standards

### The student is expected to:

**4.2A** interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;

**4.2D** round whole numbers to a given place value through the hundred thousands place;

**4.2E** represent decimals, including tenths and hundredths, using concrete and visual models and money;

**4.2F** compare and order decimals using concrete and visual models to the hundredths;

**4.2G** relate decimals to fractions that name tenths and hundredths;

**4.2H** determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line;

**4.3C** determine if two given fractions are equivalent using a variety of methods;

**4.4B** determine products of a number and 10 or 100 using properties of operations and place value understandings;

**4.8A** identify relative sizes of measurement units within the customary and metric systems;

**4.8B** convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table;

**4.8C** solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.

[2]The focus in this module is on the metric system to reinforce place value and writing measurements using mixed units. This standard is addressed again in later modules.

## Focus Mathematical Process Standards

**The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**

**MPS(E)** create and use representations to organize, record, and communicate mathematical ideas;

**MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas;

**MPS(G)** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written aor oral communication.

## Overview of Module Topics and Lesson Objectives

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.2A**<br>**5.7**<br>4.2 | 1.A<br>1.B<br>1.C<br>1.E<br>1.F<br>2.C<br>2.D<br>2.E<br>3.D<br>4.B | A | **Multiplicative Patterns on the Place Value Chart**<br>Lesson 1: Use place value patterns to understand the thousandths place.<br>Lesson 2: Use place value understanding to reason abstractly about values of digits in decimal fractions.<br>Lesson 3: Use place value understanding to convert metric units. | 3 |
| **5.2A**<br>**5.2B** | 2.C<br>2.D<br>2.E<br>2.H<br>2.I<br>3.D<br>3.E<br>4.B | B | **Decimal Fractions and Place Value Patterns**<br>Lesson 4: Name decimal fractions in expanded notation, unit form, and word form by applying place value reasoning.<br>Lesson 5: Compare decimal fractions to the thousandths using like units, and express comparisons with >, <, =. | 2 |
| **5.2C** | 2.C<br>2.D<br>2.E<br>2.G<br>3.E<br>3.F<br>4.B<br>4.I | C | **Place Value and Rounding Decimal Fractions**<br>Lessons 6–7: Round a given decimal to any place using place value understanding and the vertical number line. | 2 |

| TEKS | ELPS | Topics and Objectives | | Days |
|---|---|---|---|---|
| | | | Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day) | 2 |
| **5.3K**<br>5.2A | 2.E<br>2.G<br>2.H<br>2.I<br>3.E<br>3.F<br>4.B<br>5.F<br>5.G | D | **Adding and Subtracting Decimals**<br>Lesson 8: Add decimals using place value strategies, and relate those strategies to a written method.<br>Lesson 9: Subtract decimals using place value strategies, and relate those strategies to a written method. | 2 |
| **5.3A**<br>**5.3D**<br>**5.3E**<br>5.2A | 2.C<br>2.E<br>2.H<br>2.I<br>3.D<br>3.E<br>4.D<br>4.G<br>5.F | E | **Multiplying Decimals**<br>Lesson 10: Multiply a decimal fraction by single-digit whole numbers, relate to a written method through application of the area model and place value understanding, and explain the reasoning used.<br>Lesson 11: Multiply a decimal fraction by single-digit whole numbers, including using estimation to confirm the placement of the decimal point. | 2 |
| **5.3F**<br>**5.3G**<br>5.3K | 2.E<br>2.G<br>2.H<br>2.I<br>3.C<br>3.D<br>3.E<br>3.H<br>4.G | F | **Dividing Decimals**<br>Lesson 12: Divide decimals by single-digit whole numbers involving easily identifiable multiples using place value understanding and relate to a written method.<br>Lesson 13: Divide decimals with a remainder using place value understanding and relate to a written method.<br>Lesson 14: Divide decimals using place value understanding, including remainders in the smallest unit.<br>Lesson 15: Solve word problems using decimal operations. | 4 |
| | | | End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further applications 1 day) | 2 |
| **Total Number of Instructional Days** | | | | **19** |

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# GRADE 5 • MODULE 2

## Multi-Digit Whole Number and Decimal Fraction Operations

## Grade 5 ● Module 2

# Multi-Digit Whole Number and Decimal Fraction Operations

## OVERVIEW

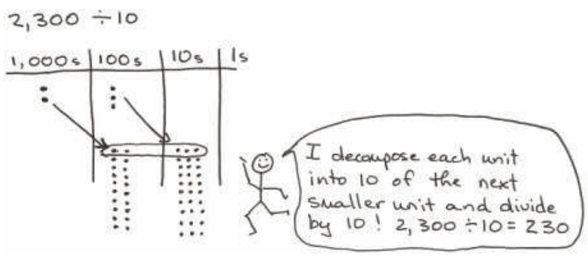
In Module 1, students explored the relationships of adjacent units on the place value chart to generalize whole number algorithms to decimal fraction operations. In Module 2, students apply the patterns of the base ten system to mental strategies and the multiplication and division algorithms.

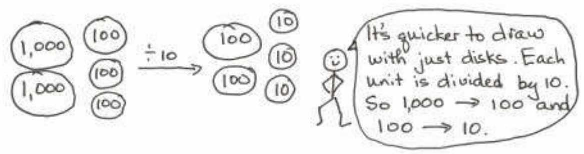
In Topic A, students explore factors, multiples, and prime and composite numbers within 100 (**5.4A**), gaining valuable insights into patterns of divisibility as they test for primes and find factors and multiples.

Topics B through E provide a sequential study of multiplication. To link to prior learning and set the foundation for understanding the standard multiplication algorithm, students begin at the concrete–pictorial level in Topic B **(5.3B)**. They use place value disks to model multi-digit multiplication of place value units, for example, 42 × 10, 42 × 100, 42 × 1,000, leading to problems such as 42 × 30, 42 × 300, and 42 × 3,000. They then round factors in Lesson 6 and discuss the reasonableness of their products **(5.3A)**. Throughout Topic B, students evaluate and write simple expressions to record their calculations using the associative property and parentheses to record the relevant order of calculations.

In Topic C, place value understanding moves toward understanding the distributive property via area models, which are used to generate and record the partial products **(5.4E, 5.4F)** of the standard algorithm **(5.3B)**. Topic D moves students from whole numbers to multiplication with decimals, again using place value as a guide to reason and make estimations about products **(5.3A, 5.3D, 5.3E)**. In Topic E, students explore multiplication as a method for expressing equivalent measures. For example, they multiply to convert between meters and centimeters or ounces and cups with measurements in both whole number and decimal form **(5.7)**.

Topics F through I provide a similar sequence for division. Topic F begins concretely with place value disks as an introduction to division with multi-digit whole numbers **(5.3A, 5.3C)**.




In the same lesson, 420 ÷ 60 is interpreted as 420 ÷ 10 ÷ 6. Next, students round dividends and two-digit divisors to nearby multiples of 10 in order to estimate single-digit quotients (e.g., 431 ÷ 58 ≈ 420 ÷ 60 = 7) and then multi-digit quotients. This work is done horizontally, outside the context of the written vertical method.

The series of lessons in Topic G lead students to divide multi-digit dividends by two-digit divisors using the written vertical method. Each lesson moves to a new level of difficulty with a sequence beginning with divisors that are multiples of 10 to non-multiples of 10. Two instructional days are devoted to single-digit quotients with and without remainders before progressing to two- and three-digit quotients **(5.3C)**.

In Topic H, students use their understanding to divide decimals by two-digit divisors in a sequence similar to that of Topic G with whole numbers **(5.3F, 5.3G)**. In Topic I, students apply the work of the module to solve multi-step word problems using multi-digit division with unknowns representing either the group size or number of groups. In this topic, an emphasis on checking the reasonableness of their answers draws on skills learned throughout the module, including refining their knowledge of place value, rounding, and estimation.

## Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Depending on students' strengths, consider consolidating Lessons 9 and 10. In Lesson 9, omit Problem 1 of the Concept Development, and move directly into renaming with the algorithm after Problem 2. Use the Problem Set from Lesson 10 for independent student practice. Consider consolidating Lessons 11 and 12 as well. Ask students to estimate the product beginning with the Concept Development of Lesson 11, and then use the Problem Set from Lesson 12 for student practice. Similarly, Lessons 15 and 16 can also be consolidated. Use estimation from the outset, and have students practice with the Problem Set from Lesson 16.

It is not recommended to omit any lessons from Topic E as it is a foundation for work later in the year. Students convert measurement units from small to large and from large to small using multiplication. This significantly expedites their understanding of and fluency with conversion as the year continues.

# Focus Grade Level Standards

## Number and Operations

**The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:**

**5.3A** estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division;

**5.3B** multiply with fluency a three-digit number by a two-digit number using the standard algorithm;

**5.3C** solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor using strategies and the standard algorithm;

**5.3D** represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;

**5.3E** solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;

**5.3F** represent quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using objects and pictorial models, including area models;

**5.3G** solve for quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm.

## Algebraic Reasoning

**The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:**

**5.4A** identify prime and composite numbers;

**5.4B** represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;

**5.4E** describe the meaning of parentheses and brackets in a numeric expression;

**5.4F** simplify numerical expressions that do not involve exponents, including up to two levels of grouping.

## Geometry and Measurement

**The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to:**

**5.7** solve problems by calculating conversions within a measurement system, customary or metric.

## Foundational Standards

**The student is expected to:**

**3.5C** describe a multiplication expression as a comparison such as 3 × 24 represents 3 times as much as 24;

**4.2A** interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;

**4.4A** add and subtract whole numbers and decimals to the hundredths place using the standard algorithm;

**4.4C** represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15;

**4.4D** use strategies and algorithms, including the standard algorithm, to multiply up to a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

**4.4E** represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;

**4.4F** use strategies and algorithms, including the standard algorithm, to divide up to a four-digit dividend by a one-digit divisor;

**4.4G** round to the nearest 10, 100, or 1,000 or use compatible numbers to estimate solutions involving whole numbers;

**4.5A** represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity.

## Focus Mathematical Process Standards

**The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**

**MPS(A)** apply mathematics to problems arising in everyday life, society, and the workplace;

**MPS(B)** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

**MPS(D)** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

**MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas.

## Overview of Module Topics and Lesson Objectives

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.4A** | 1.A<br>1.C<br>1.E<br>2.C<br>2.E<br>2.I<br>3.E<br>4.G<br>5.G | A | **Prime and Composite Numbers**<br>Lesson 1: Find factor pairs for numbers to 100, and use understanding of factors to define prime and composite.<br>Lesson 2: Use division and the associative property to test for factors and observe patterns.<br>Lesson 3: Determine if a whole number is a multiple of another number.<br>Lesson 4: Explore properties of prime and composite numbers to 100 by using multiples. | 4 |
| **5.3A**<br>**5.3B**<br>5.4F | 1.C<br>2.I<br>3.C<br>3.D<br>4.J<br>5.G | B | **Mental Strategies for Multi-Digit Whole Number Multiplication**<br>Lesson 5: Multiply multi-digit whole numbers and multiples of 10 using place value patterns and the distributive and associative properties.<br>Lesson 6: Estimate multi-digit products by rounding factors to a basic fact and using place value patterns. | 2 |
| **5.3A**<br>**5.3B**<br>**5.4B**<br>**5.4E**<br>**5.4F** | 1.C<br>2.C<br>2.E<br>2.G<br>2.I<br>3.E<br>3.H<br>4.G<br>5.G | C | **The Standard Algorithm for Multi-Digit Whole Number Multiplication**<br>Lesson 7: Write and interpret numerical expressions, and compare expressions using a visual model.<br>Lesson 8: Convert numerical expressions into unit form as a mental strategy for multi-digit multiplication.<br>Lesson 9: Connect visual models and the distributive property to partial products of the standard algorithm without renaming.<br>Lessons 10–11: Connect area models and the distributive property to partial products of the standard algorithm with renaming.<br>Lesson 12: Fluently multiply multi-digit whole numbers using the standard algorithm and using estimation to check for reasonableness of the product.<br>Lesson 13: Fluently multiply multi-digit whole numbers using the standard algorithm to solve multi-step word problems. | 7 |

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.3A**<br>**5.3D**<br>**5.3E**<br>5.4B<br>5.4E<br>5.4F | 1.C<br>1.H<br>2.H<br>2.I<br>3.F<br>4.K<br>5.G | D | **Decimal Multi-Digit Multiplication**<br>Lesson 14: Multiply decimal fractions with tenths by multi-digit whole numbers using place value understanding to record partial products.<br>Lesson 15: Multiply decimal fractions by multi-digit whole numbers through conversion to a whole number problem and reasoning about the placement of the decimal.<br>Lesson 16: Reason about the product of a whole number and a decimal with hundredths using place value understanding and estimation. | 3 |
| **5.3B**<br>**5.3D**<br>**5.3E**<br>**5.7** | 2.C<br>2.E<br>2.G<br>2.I<br>3.E<br>4.J<br>5.D<br>5.G | E | **Measurement Word Problems with Whole Number and Decimal Multiplication**<br>Lesson 17: Use whole number multiplication to express equivalent measurements.<br>Lesson 18: Use decimal multiplication to express equivalent measurements.<br>Lesson 19: Solve two-step word problems involving measurement conversions. | 3 |
| | | | Mid-Module Assessment: Topics A–E (assessment ½ day, return ½ day, remediation or further applications 2 days) | 3 |
| **5.3A**<br>**5.3C**<br>5.2A | 2.G<br>3.E<br>3.G<br>3.H<br>4.K<br>5.G | F | **Mental Strategies for Multi-Digit Whole Number Division**<br>Lesson 20: Use *divide by 10* patterns for multi-digit whole number division.<br>Lessons 21–22: Use compatible numbers to approximate quotients with two-digit divisors. | 3 |
| **5.3C**<br>5.4B | 1.A<br>2.E<br>2.G<br>2.I<br>3.E<br>3.G<br>3.H<br>4.K<br>5.G | G | **Partial Quotients and Multi-Digit Whole Number Division**<br>Lesson 23: Divide two- and three-digit dividends by multiples of 10 with single-digit quotients, and make connections to a written method.<br>Lessons 24–25: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients, and make connections to a written method.<br>Lessons 26–27: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients, reasoning about the decomposition of successive remainders in each place value. | 5 |

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.3F**<br>**5.3G**<br>5.2A<br>5.4B | 2.E<br>2.G<br>2.I<br>4.J<br>5.G | H | **Partial Quotients and Multi-Digit Decimal Division**<br>Lesson 28: Divide decimal dividends by multiples of 10, reasoning about the placement of the decimal point and making connections to a written method.<br>Lesson 29: Use basic facts to approximate decimal quotients with two-digit divisors, reasoning about the placement of the decimal point.<br>Lessons 30–31: Divide decimal dividends by two-digit divisors, estimating quotients, reasoning about the placement of the decimal point, and making connections to a written method. | 4 |
| **5.3C**<br>**5.3F**<br>**5.3G**<br>**5.4B** | 1.H<br>2.E<br>3.E<br>3.G<br>4.K<br>5.G | I | **Measurement Word Problems with Multi-Digit Division**<br>Lessons 32–33: Solve division word problems involving multi-digit division with group size unknown and the number of groups unknown. | 2 |
| | | | End-of-Module Assessment: Topics A–I (assessment 1 day, return 1 day, remediation or further application 2 days) | 4 |
| **Total Number of Instructional Days** | | | | **40** |

# Terminology

## New or Recently Introduced Terms

- Composite number (positive integer having three or more whole number factors)
- Conversion factor (the factor in a multiplication sentence that renames one measurement unit as another equivalent unit, e.g., 14 × (1 in) = 14 × ($\frac{1}{12}$ ft); 1 in and $\frac{1}{12}$ ft are the conversion factors)
- Decimal fraction (a proper fraction whose denominator is a power of 10)
- Multiplier (a quantity by which a given number—a multiplicand—is to be multiplied)
- Parentheses (the symbols used to relate order of operations)
- Prime number (positive integer greater than 1 having whole number factors of only 1 and itself)

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# GRADE 5 • MODULE 3

## Addition and Subtraction of Fractions

## Grade 5 • Module 3

# Addition and Subtraction of Fractions

## OVERVIEW

In Module 3, students' understanding of addition and subtraction of fractions extends from earlier work with fraction equivalence and decimals. This module marks a significant shift away from the elementary grades' centrality of base ten units to the study and use of the full set of fractional units from Grade 5 forward, especially as applied to algebra.

In Topic A, students see that fraction addition and subtraction are analogous to whole number addition and subtraction. Students add and subtract fractions with unlike denominators (**5.3.H, 5.3K**) by replacing different fractional units with an equivalent fraction or like unit.

1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths

$$\frac{1}{4}+\frac{2}{3}=\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$$

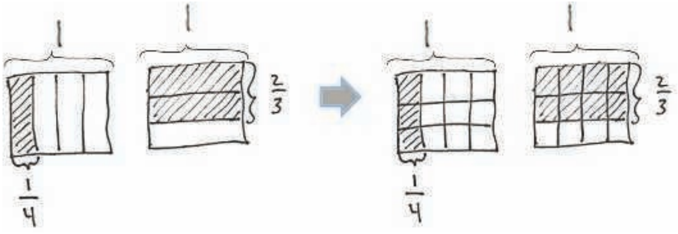
This is not a new concept, but certainly a new level of complexity. Students have added equivalent or like units since kindergarten, adding frogs to frogs, ones to ones, tens to tens, etc.

1 boy + 2 girls = 1 child + 2 children = 3 children

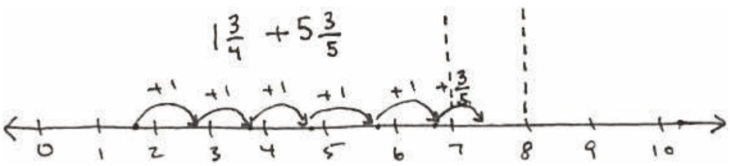
1 liter – 375mL = 1,000 mL - 375 mL = 625 mL

Throughout the module, a concrete to pictorial to abstract approach is used to convey this simple concept. Topic A primarily uses the rectangular fractional model because it is useful for creating smaller like units by means of partitioning (e.g., thirds and fourths are changed to twelfths to create equivalent fractions as in the diagram below). In Topic B, students move away from the pictorial altogether as they are empowered to write equations clarified by the model.

$$\frac{1}{4}+\frac{2}{3}=\left(\frac{1\times3}{4\times3}\right)+\left(\frac{2\times4}{3\times4}\right)=\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$$



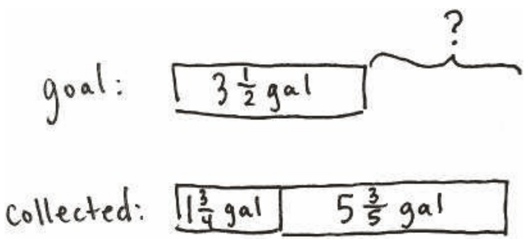
Topic B also uses the number line when adding and subtracting fractions greater than or equal to 1 so that students begin to see and manipulate fractions in relation to larger whole numbers and to each other. The number line allows students to pictorially represent larger whole numbers. For example, "Between which two whole numbers does the sum of $1\frac{3}{4}$ and $5\frac{3}{5}$ lie?"




$\_\_\_\_ < 1\frac{3}{4} + 5\frac{3}{5} < \_\_\_\_$

This leads to an understanding of and skill with solving more complex problems, which are often embedded within multi-step word problems:

> *Cristina and Matt's goal is to collect a total of* $3\frac{1}{2}$ *gallons of sap from the maple trees. Cristina collected* $1\frac{3}{4}$ *gallons. Matt collected* $5\frac{3}{5}$ *gallons. By how much did they beat their goal?*




$$1\frac{3}{4} + 5\frac{3}{5} - 3\frac{1}{2} = 3 + \left(\frac{3 \times 5}{4 \times 5}\right) + \left(\frac{3 \times 4}{5 \times 4}\right) - \left(\frac{1 \times 10}{2 \times 10}\right)$$

$$= 3 + \frac{15}{20} + \frac{12}{20} - \frac{10}{20} = 3\frac{17}{20}$$

Cristina and Matt beat their goal by $3\frac{17}{20}$ gallons.

Word problems are a part of every lesson. Students are encouraged to draw strip diagrams, which encourage them to recognize part–whole relationships with fractions that they have seen with whole numbers since Grade 1.

In Topic C, students strategize to solve multi-term problems and more intensely assess the reasonableness of their solutions to equations and word problems with fractional units (**5.3H**).

> *"I know my answer makes sense because the total amount of sap they collected is about 7 and a half gallons. Then, when we subtract 3 gallons, that is about 4 and a half. Then, 1 half less than that is about 4.* $3\frac{17}{20}$ *is just a little less than 4."*

The Mid-Module Assessment follows Topic A. The End-of-Module Assessment follows Topic C.

## Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. In Lesson 1, omit the paper folding exercise, and consider it a remediation tool. Omit the Sprint in Lesson 10, and replace it with simple reasoning about fractions on the number line, such as "Is $\frac{3}{4}$ greater than or less than $\frac{1}{2}$? $\frac{3}{5}$? $\frac{3}{7}$?" In Lesson 13, choose two or three problems, and omit the others.

Use the omitted problems as Application Problems in future lessons. Consider omitting Lesson 14 and using it in a center for early finishers, or have advanced students work the problems and present their solutions in a video or interactive demonstration. Consider asking the following questions to students, “Have you ever thought about what the whole would look like if this paper were one-half? What if it were one-third? What if this is three-fourths of the whole? What would the whole look like then?”

Note: In the first year of implementation, beginning in Lesson 3, be sure to include the fluency activities requiring students to subtract fractions less than one from a whole number (e.g., $4-\frac{5}{8}$) in order to prepare students to subtract larger mixed numbers in Topic B. Model these fluency activities on the number line and with a strip diagram.

## Focus Grade Level Standards

### Number and Operations

**The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:**

**5.3H** represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations;

**5.3K** add and subtract positive rational numbers fluently.

## Foundational Standards

**The student is expected to:**

**4.3A** represent a fraction *a/b* as a sum of fractions 1/*b*, where *a* and *b* are whole numbers and *b* > 0, including when *a* > *b*;

**4.3B** decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations;

**4.3C** determine if two given fractions are equivalent using a variety of methods;

**4.3E** represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations;

**4.3F** evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole;

**4.3G** represent fractions and decimals to the tenths or hundredths as distances from zero on a number line.

# Focus Mathematical Process Standards

**The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**

**MPS(A)** apply mathematics to problems arising in everyday life, society, and the workplace

**MPS(B)** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

**MPS(C)** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

**MPS(G)** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

# Overview of Module Topics and Lesson Objectives

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.3H**<br>**5.3K** | 1.E<br>1.H<br>2.E<br>2.I<br>3.E<br>3.F<br>3.G<br>4.G<br>4.H<br>5.G | A | **Making Like Units Pictorially**<br>Lesson 1: Add fractions with unlike units using the strategy of creating equivalent fractions.<br>Lesson 2: Add fractions with sums between 1 and 2.<br>Lesson 3: Subtract fractions with unlike units using the strategy of creating equivalent fractions.<br>Lesson 4: Subtract fractions from numbers between 1 and 2.<br>Lesson 5: Solve two-step word problems. | 5 |
| | | | Mid-Module Assessment: Topic A (assessment ½ day, return ½ day, remediation or further applications 2 days) | 3 |
| **5.3H**<br>**5.3K** | 1.C<br>1.H<br>2.E<br>3.E<br>3.F<br>3.G<br>4.A<br>4.C<br>4.G<br>5.G | B | **Making Like Units Numerically**<br>Lesson 6: Add fractions to and subtract fractions from whole numbers using equivalence and the number line as strategies.<br>Lesson 7: Add fractions making like units numerically.<br>Lesson 8: Add fractions with sums greater than 2.<br>Lesson 9: Subtract fractions making like units numerically.<br>Lesson 10: Subtract fractions greater than or equal to 1. | 5 |
| **5.3H**<br>**5.3K** | 1.C<br>1.H<br>3.C<br>3.E<br>3.H<br>4.B<br>4.K<br>5.G | C | **Further Applications**<br>Lesson 11: Use fraction benchmark numbers to assess reasonableness of addition and subtraction equations.<br>Lesson 12: Strategize to solve multi-term problems.<br>Lesson 13: Solve multi-step word problems; assess reasonableness of solutions using benchmark numbers.<br>Lesson 14: Explore part-to-whole relationships. | 4 |
| | | | End-of-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 2 days) | 3 |
| **Total Number of Instructional Days** | | | | **20** |

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# GRADE 5 • MODULE 4

## Multiplication and Division of Fractions

Grade 5 • Module 4

# Multiplication and Division of Fractions

## OVERVIEW

In Module 4, students learn to multiply fractions and begin working with fraction division.

Students begin Topic A by decomposing non-unit fractions and representing these decompositions as repeated addition sentences. They build on this concept by representing addition sentences as multiplication, just as they did in Grade 4: $\frac{3}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=3\times\frac{1}{4}$. They will apply this understanding of decomposition and multiplication and use the associative property in order to multiply a whole number by a fraction (**5.3I**). In using the associative property, student multiply and then consider the unit.

$$4\times\frac{2}{3}=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=4\times2\text{ thirds}=(4\times2)\text{ thirds}=8\text{ thirds}=\frac{8}{3}$$

$$4\times\frac{2}{3}=\frac{(4\times2)}{3}=\frac{8}{3}$$

$$n\times\frac{a}{b}=\frac{n\times a}{b}$$

Continuing in Topic A, students proceed to multiplying a whole number by a mixed unit number by applying the distributive property. They recognize that they are multiplying each part of a mixed number by the whole number, and notice how efficient it is to use the distributive property. The topic closes with solving multiplicative comparison word problems involving fractions, and interpretation of data presented in dot plots (**5.9C**).

$$5\times3\frac{3}{4}=5\times\left(3+\frac{3}{4}\right)$$

$$=(5\times3)+\left(5\times\frac{3}{4}\right)$$

$$=15+\frac{15}{4}$$

$$=15+3\frac{3}{4}$$

$$=18\frac{3}{4}$$

In Topic B, students interpret finding a fraction of a set ($\frac{3}{4}$ of 24) as multiplication of a whole number by a fraction ($\frac{3}{4} \times 24$) and use strip diagrams to support their understandings (**5.3I**). This, in turn, leads students to see division by a whole number as being equivalent to multiplication by its reciprocal. That is, division by 2, for example, is the same as multiplication by $\frac{1}{2}$. Students also use the commutative property to relate a fraction of a set to the Topic A repeated addition interpretation of multiplication by a fraction. This offers opportunities for students to reason about various strategies for multiplying fractions and whole numbers. Students apply their knowledge of a fraction of a set and previous conversion experiences (with scaffolding from a conversion chart, if necessary) to find a fraction of a measurement, thus converting a larger unit to an equivalent smaller unit (e.g., $\frac{1}{3}$ minutes = 20 seconds and $2\frac{1}{4}$ feet = 27 inches) (**5.7**).

Interpreting numerical expressions opens Topic C as students learn to evaluate expressions with parentheses, such as $3 \times (\frac{2}{3} - \frac{1}{5})$ or $\frac{2}{3} \times (7 + 9)$ (**5.4F**). They then learn to interpret numerical expressions, such as 3 *times the difference between* $\frac{2}{3}$ *and* $\frac{1}{5}$ or *two-thirds the sum of 7 and 9* (**5.4E**). Students generate word problems that lead to the same calculation (**5.3I**), such as "Kelly combined 7 ounces of carrot juice and 5 ounces of orange juice in a glass. Jack drank $\frac{2}{3}$ of the mixture. How much did Jack drink?" Solving word problems (**5.3I**) allows students to apply new knowledge of fraction multiplication in context, and strip diagrams are used to model multi-step problems requiring the use of addition, subtraction, and multiplication of fractions.

Topic D begins the work of division with fractions. Students use strip diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number (**5.3J, 5.3L**). Using the same thinking developed in Module 2 to divide whole numbers, students reason about how many *fourths* are in 5 when considering such cases as $5 \div \frac{1}{4}$. They also reason about the size of the unit when $\frac{1}{4}$ is partitioned into 5 equal parts: $\frac{1}{4} \div 5$.

In Topic E, students engage in activities designed to bring the Personal Financial Literacy standards for Grade 5 to life. Through the use of a consistent real-life scenario, students understand how to balance a simple budget. Then they differentiate between gross income and net income while defining income tax and payroll tax. This gives students an opportunity to use what they know about fraction of a set to do simple tax calculations (**5.10A**). Students understand the difference between sales tax and property tax and discuss the advantages and disadvantages of various forms of payment (**5.10C, 5.10E**).

The module concludes with Topic F, in which numerical expressions involving fraction multiplication are interpreted and evaluated (**5.4E, 5.4F**). Students create and solve word problems involving both multiplication and division of fractions and decimal fractions.

The Mid-Module Assessment is administered after Topic C, and the End-of-Module Assessment follows Topic F.

### Notes on Pacing for Differentiation

Lessons 13 and 14 are both word problem lessons involving addition, subtraction, and multiplication with fractions. Omit Lesson 13, but include Problems 1 and 4 as part of Lesson 14. In Lesson 14, use Problems 4 and 5 as an extension or challenge for early finishers, and omit Problems 5 and 6 from the Homework.

Note: Looking ahead, Topic D of Module 5 includes drawing in 5 of the 6 geometry lessons. These drawings with the protractor are critical to the coherence of the geometry standards of Grade 4 and those of middle school. These drawings could be completed during Module 4 but at a different time of the day, such as art class or for morning work. It is best that drawing with the protractor be taught by the math teacher. This modification allows for the later consolidation of Lessons 16, 17, 18, and 19 in Module 5.

## Focus Grade Level Standards

### Number and Operations

**The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:**

**5.3I** represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models;

**5.3J** represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1/3 \div 7$ and $7 \div 1/3$ using objects and pictorial models, including area models;

**5.3L** divide whole numbers by unit fractions and unit fractions by whole numbers.

### Algebraic Reasoning

**The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:**

**5.4E** describe the meaning of parentheses and brackets in a numeric expression;

**5.4F** simplify numerical expressions that do not involve exponents, including up to two levels of grouping.

## Geometry and Measurement

**The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to:**

**5.7** solve problems by calculating conversions within a measurement system, customary or metric.

## Data Analysis

**The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:**

**5.9A** represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-and-leaf plots;

**5.9C** solve one- and two-step problems using data from a frequency table, dot plot, bar graph, stem-and-leaf plot, or scatterplot.

## Personal Financial Literacy

**The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:**

**5.10A** define income tax, payroll tax, sales tax, and property tax;

**5.10B** explain the difference between gross income and net income;

**5.10C** identify the advantages and disadvantages of different methods of payment, including check, credit card, debit card, and electronic payments;

**5.10D** develop a system for keeping and using financial records;

**5.10E** describe actions that might be taken to balance a budget when expenses exceed income;

**5.10F** balance a simple budget.

# Foundational Standards

**The student is expected to:**

**4.2E** represent decimals, including tenths and hundredths, using concrete and visual models and money;

**4.2G** relate decimals to fractions that name tenths and hundredths;

**4.2H** determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line;

**4.3A** represent a fraction a/b as a sum of fractions 1/b, where a and b are whole numbers and b > 0, including when a > b;

**4.3B** decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations;

**4.3C** determine if two given fractions are equivalent using a variety of methods;

**4.3D** compare two fractions with different numerators and different denominators and represent the comparison using the symbols >, =, or <;

**4.3E** represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations;

**4.3F** evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole;

**4.3G** represent fractions and decimals to the tenths or hundredths as distances from zero on a number line;

**4.8B** convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table.

# Focus Mathematical Process Standards

**The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**

**MPS(C)** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

**MPS(D)** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

**MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas.

## Overview of Module Topics and Lesson Objectives

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.3I**<br>**5.9A**<br>**5.9C**<br>5.3K | 1.E<br>1.H<br>2.E<br>2.I<br>3.E<br>3.G<br>3.H<br>4.E<br>4.G<br>5.G | A | **Repeated Addition of Fractions as Multiplication**<br>Lesson 1: Decompose non-unit fractions and represent them as a whole number times a unit fraction using strip diagrams.<br>Lessons 2–3: Represent the multiplication of n times $a/b$ as $(n \times a)/b$ using the associative property and visual models.<br>Lessons 4–5: Find the product of a whole number and a mixed number using the distributive property.<br>Lesson 6: Solve multiplicative comparison word problems involving fractions.<br>Lesson 7: Solve word problems involving the multiplication of a whole number and a fraction including those involving dot plots. | 7 |
| **5.3I**<br>**5.7**<br>5.3K | 1.C<br>2.E<br>2.I<br>3.C<br>3.E<br>3.H<br>4.G<br>5.G | B | **Multiplication of a Whole Number by a Fraction**<br>Lesson 8: Find a fraction of a set concretely and pictorially.<br>Lesson 9: Multiply any whole number by a fraction using strip diagrams.<br>Lesson 10: Relate a fraction of a set to the repeated addition interpretation of fraction multiplication.<br>Lesson 11: Find a fraction of a measurement, and solve word problems. | 4 |
| **5.3I**<br>**5.4E**<br>**5.4F**<br>**5.7**<br>5.3K | 1.C<br>1.H<br>2.E<br>3.D<br>3.E<br>3.H<br>4.B<br>4.F<br>4.K<br>5.C<br>5.G | C | **Fraction Expressions and Word Problems**<br>Lesson 12: Compare and evaluate expressions with parentheses and brackets.<br>Lessons 13–14: Solve and create fraction word problems involving addition, subtraction, and multiplication.<br>Lesson 15: Convert measures involving whole numbers, and solve multi-step word problems.<br>Lesson 16: Convert mixed unit measurements, and solve multi-step word problems. | 5 |
| | | | Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 2 days) | 3 |

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.3J**<br>**5.3L**<br>5.3A | 1.C<br>1.E<br>2.E<br>2.I<br>3.E<br>3.H<br>4.G<br>5.G | D | **Division of Fractions**<br>Lesson 17: Divide a whole number by a unit fraction.<br>Lesson 18: Divide a unit fraction by a whole number.<br>Lesson 19: Solve problems involving fraction division.<br>Lesson 20: Write equations and word problems corresponding to strip and number line diagrams. | 4 |
| **5.10A**<br>**5.10B**<br>**5.10C**<br>**5.10D**<br>**5.10E**<br>**5.10F**<br>5.3E<br>5.3K | 2.B<br>2.C<br>2.E<br>2.G<br>2.I<br>3.B<br>3.E<br>3.G<br>4.J<br>5.G | E | **Applying Fraction and Decimal Multiplication to Personal Financial Literacy**<br>Lesson 21: Balance a simple budget.<br>Lesson 22: Explain the difference between gross income and net income. Define income tax and payroll tax.<br>Lesson 23: Define property tax and sales tax.<br>Lesson 24: Identify the advantages and disadvantages of different methods of payment. | 4 |
| **5.4E**<br>**5.4F** | 2.E<br>2.I<br>3.E<br>4.K<br>5.G | F | **Interpretation of Numerical Expressions**<br>Lesson 25: Interpret and evaluate numerical expressions. | 1 |
| | | | End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further applications 2 days) | 3 |
| **Total Number of Instructional Days** | | | | **31** |

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# GRADE 5 • MODULE 5

## Addition and Multiplication with Volume and Area

## Grade 5 • Module 5

# Addition and Multiplication with Volume and Area

## OVERVIEW

In this 28-day module, students work with two- and three-dimensional figures. Volume is introduced to students through concrete exploration of cubic units and culminates with the development of the volume formula for right rectangular prisms. The second half of the module turns to extending students' understanding of two-dimensional figures. Students combine prior knowledge of area with newly acquired knowledge of fraction multiplication to determine the area of rectangular figures with fractional side lengths. They then engage in hands-on construction of two-dimensional shapes, developing a foundation for classifying the shapes by reasoning about their attributes. This module fills a gap between Grade 4's work with two-dimensional figures and Grade 6's work with volume and area.

In Topic A, students extend their spatial structuring to three dimensions through an exploration of volume. Students come to see volume as an attribute of solid figures and understand that cubic units are used to measure it (**5.6A**). Using improvised, customary, and metric units, they build three-dimensional shapes, including right rectangular prisms, and count units to find the volume (**5.6A**). By developing a systematic approach to counting the unit cubes, students make connections between area and volume. They partition a rectangular prism into layers of unit cubes and reason that the number of unit cubes in a single layer corresponds to the number of unit squares on a face. They begin to conceptualize the layers themselves, oriented in any one of three directions, as iterated units. This understanding allows students to reason about containers formed by box templates and nets, reasonably predict the number of cubes required to fill them, and test their predictions by packing the containers.

Concrete understanding of volume and multiplicative reasoning come together in Topic B as the systematic counting from Topic A leads naturally to formulas for finding the volume of a right rectangular prism and the special form of that formula for finding the volume of cubes (**5.4G, 5.4H, 5.6B**). Students solidify the connection between volume as *packing* and volume as *filling* by comparing the amount of liquid that fills a container to the number of cubes that can be packed into it. This connection is formalized as students see that 1 cubic centimeter is equal to 1 milliliter. Complexity increases as students use their knowledge that volume is additive to partition and calculate the total volume of solid figures composed of non-overlapping, rectangular prisms. While TEKS does not specify that students understand volume as additive, this shift in complexity provides multiple opportunities for students to apply the formulas for volume. Word problems involving the volume of rectangular prisms with whole number edge lengths solidify understanding and give students the opportunity to reason about multiplication in the context of volume. Topic B concludes with a design project that gives students the opportunity to apply the concepts and formulas they have learned throughout Topics A and B to create a sculpture of a specified volume composed of varied rectangular prisms with parameters given in the project description.

In Topic C, students extend their understanding of area as they use rulers and set squares to construct and measure rectangles with fractional side lengths and find their areas. Students apply their extensive knowledge of fraction multiplication to interpret areas of rectangles with fractional side lengths (**5.4H**) and solve real-world problems involving these figures (**5.3I**). Visual models and equations are used to represent the problems through the Read-Draw-Write (RDW) protocol.

In Topic D, students draw two-dimensional shapes to analyze their attributes and use those attributes to classify them. Familiar figures, such as triangles, parallelograms, rhombuses, squares, trapezoids, etc., have all been defined in earlier grades and, in Grade 4, students have gained an understanding of shapes beyond the intuitive level and have classified triangles by angle measure. Grade 5 extends this understanding through classification of triangles by side length and an in-depth analysis of the properties and defining attributes of quadrilaterals. Grade 4's work with the protractor is applied to construct various quadrilaterals. Using measurement tools illuminates the attributes used to define and recognize each quadrilateral (**5.5**). Students see, for example, that the same process they used to construct a parallelogram also produces a rectangle when all angles are constructed to measure 90°. Students then analyze defining attributes and create a hierarchical classification of quadrilaterals (**5.5**).

## Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Omit Lessons 8 and 9, in which students create sculptures out of multiple rectangular prisms. Instead, consider asking the art teacher to complete a similar project with students. Lessons 13 and 14 can be consolidated since they share the same objective. Use Problems 1 and 2 from Lesson 13 and Problems 1 and 2 from Lesson 14. Problem 3 from Lesson 14 can be an extension for early finishers. Omit Lesson 22, and instead, use it in a center or periodically as morning work.

# Focus Grade Level Standards

## Number and Operations

**The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:**

**5.3I** represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models.

## Algebraic Reasoning

**The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:**

**5.4G** use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube ($V = l \times w \times h$, $V = s \times s \times s$, and $V = Bh$);

**5.4H** represent and solve problems related to perimeter and/or area and related to volume.

## Geometry and Measurement

**The student applies mathematical process standards to classify two-dimensional figures by attributes and properties. The student is expected to:**

**5.5** classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties.

**The student applies mathematical process standards to understand, recognize, and quantify volume. The student is expected to:**

**5.6A** recognize a cube with side length of one unit as a unit cube having one cubic unit of volume and the volume of a three-dimensional figure as the number of unit cubes ($n$ cubic units) needed to fill it with no gaps or overlaps if possible;

**5.6B** determine the volume of a rectangular prism with whole number side lengths in problems related to the number of layers times the number of unit cubes in the area of the base.

## Foundational Standards

**The student is expected to:**

| | |
|---|---|
| **3.6A** | classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language; |
| **3.6B** | use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories; |
| **3.8A** | summarize a data set with multiple categories using a frequency table, dot plot, pictograph, or bar graph with scaled intervals; |
| **3.8B** | solve one- and two-step problems using categorical data represented with a frequency table, dot plot, pictograph, or bar graph with scaled intervals; |
| **4.5C** | use models to determine the formulas for the perimeter of a rectangle ($l + w + l + w$ or $2l + 2w$), including the special form for perimeter of a square (4s) and the area of a rectangle ($l \times w$); |
| **4.5D** | solve problems related to perimeter and area of rectangles where dimensions are whole numbers; |
| **4.6A** | identify points, lines, line segments, rays, angles, and perpendicular and parallel lines; |
| **4.6C** | apply knowledge of right angles to identify acute, right, and obtuse triangles; |
| **4.6D** | classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size; |
| **4.7A** | illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is “cut out” by the rays of the angle. Angle measures are limited to whole numbers; |
| **4.7B** | illustrate degrees as the units used to measure an angle, where 1/360 of any circle is one degree and an angle that “cuts” $n$/360 out of any circle whose center is at the angle’s vertex has a measure of $n$ degrees. Angle measures are limited to whole numbers; |
| **4.7C** | determine the approximate measures of angles in degrees to the nearest whole number using a protractor; |
| **4.7D** | draw an angle with a given measure; |
| **4.7E** | determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures. |

## Focus Mathematical Process Standards

**The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**

- **MPS(A)** apply mathematics to problems arising in everyday life, society, and the workplace;
- **MPS(B)** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- **MPS(D)** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- **MPS(E)** create and use representations to organize, record, and communicate mathematical ideas;
- **MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas;
- **MPS(G)** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

## Overview of Module Topics and Lesson Objectives

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.6A**<br>**5.6B** | 1.C<br>1.F<br>2.E<br>2.I<br>3.J<br>4.I<br>5.G | A | **Concepts of Volume**<br>Lesson 1: Explore volume by building with and counting unit cubes.<br>Lesson 2: Find the volume of a right rectangular prism by packing with cubic units and counting.<br>Lesson 3: Compose and decompose right rectangular prisms using layers. | 3 |
| **5.4G**<br>**5.4H**<br>**5.6A**<br>**5.6B** | 1.E<br>1.H<br>2.C<br>2.E<br>2.H<br>3.E<br>3.F<br>4.B<br>5.E<br>5.G | B | **Volume and the Operations of Multiplication and Addition**<br>Lesson 4: Use multiplication to calculate volume.<br>Lesson 5: Use multiplication to connect volume as *packing* with volume as *filling*.<br>Lesson 6: Find the total volume of solid figures composed of two non-overlapping rectangular prisms.<br>Lesson 7: Solve word problems involving the volume of rectangular prisms with whole number edge lengths.<br>Lessons 8–9: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters. (Optional) | 6 |
| | | | Mid-Module Assessment: Topics A–B (assessment 1 day, return 1 day, remediation or further applications 1 day) | 3 |
| **5.3I**<br>**5.4H** | 1.C<br>2.E<br>2.H<br>2.I<br>3.E<br>3.F<br>3.G<br>3.J<br>4.B<br>5.A | C | **Area of Rectangular Figures with Fractional Side Lengths**<br>Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.<br>Lesson 11: Measure to find the area of rectangles with fractional side lengths.<br>Lesson 12: Multiply mixed number factors, and relate to the distributive property and the area model.<br>Lessons 13–14: Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations. | 5 |

| TEKS | ELPS | Topics and Objectives | | | Days |
|---|---|---|---|---|---|
| **5.5** | 1.C<br>2.D<br>2.E<br>3.C<br>3.D<br>3.E<br>3.H<br>4.G<br>5.G | D | **Drawing, Analysis, and Classification of Two-Dimensional Shapes**<br>Lesson 15:<br><br>Lesson 16:<br>Lesson 17:<br><br>Lesson 18:<br><br>Lesson 19:<br><br><br>Lesson 20:<br><br>Lesson 21:<br><br>Lesson 22: | <br>Analyze and classify triangles based on side length, angle measure, or both.<br>Define and construct triangles from given criteria.<br>Draw trapezoids to clarify their attributes, and define trapezoids based on those attributes.<br>Draw parallelograms to clarify their attributes, and define parallelograms based on those attributes.<br>Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.<br>Draw kites and squares to clarify their attributes, and define kites and squares based on those attributes.<br>Classify two-dimensional figures in a hierarchy based on properties.<br>Draw and identify varied two-dimensional figures from given attributes. | 8 |
| | | | End-of-Module Assessment: Topics A–D (assessment 1 day, return 1 day, remediation or further applications 1 day) | | 3 |
| **Total Number of Instructional Days** | | | | | **28** |

# Terminology

## New or Recently Introduced Terms

- Base (one face of a three-dimensional solid—often thought of as the surface on which the solid rests)
- Bisect (divide into two equal parts)
- Cubic units (cubes of the same size used for measuring volume)
- Equilateral triangle (triangle with three equal sides)
- Height (adjacent layers of the base that form a rectangular prism)
- Hierarchy (series of ordered groupings of shapes)
- Isosceles triangle (triangle with at least two equal sides)
- Scalene triangle (triangle with no sides or angles equal)

Table of Contents

# GRADE 5 • MODULE 6

## Problem Solving with the Coordinate Plane and Data

## Grade 5 • Module 6

# Problem Solving with the Coordinate Plane and Data

## OVERVIEW

In this 42-day module, students develop a coordinate system for the first quadrant of the coordinate plane and use it to solve problems. Students use the familiar number line as an introduction to the idea of a coordinate and construct two perpendicular number lines to create a coordinate system on the plane. They see that just as points on the line can be located by their distance from 0, the plane's coordinate system can be used to locate and plot points using two coordinates. They then use the coordinate system to explore relationships between points, ordered pairs, patterns, lines and, more abstractly, the rules that generate them. This study culminates in an exploration of the coordinate plane in real-world applications.

In Topic A, students come to realize that *any* line, regardless of orientation, can be made into a number line by first locating zero, choosing a unit length, and partitioning the length-unit into fractional lengths as desired. They are introduced to the concept of a coordinate as describing the distance of a point on the line from zero. As students construct these number lines in various orientations on a plane, they explore ways to describe the position of points *not* located on the lines. This discussion leads to the discovery that a second number line, perpendicular to the first, creates an efficient, precise way to describe the location of these points (**5.8A**). Thus, points can be located using coordinate pairs, $(a,b)$, by starting at the origin, traveling a distance of $a$ units along the $x$-axis, and traveling a distance of $b$ units along a line parallel to the $y$-axis. Students describe given points using coordinate pairs and, conversely, use given coordinate pairs to plot points (**5.8B**). The topic concludes with an investigation of patterns in coordinate pairs along lines parallel to the axes, which leads to the discovery that these lines consist of the set of points whose distance from the $x$- or $y$-axis is constant.

Students move into plotting points and using them to draw lines in the plane in Topic B. They investigate patterns relating the $x$- and $y$-coordinates of the points on the line and reason about the patterns in the ordered pairs, laying important groundwork for Grade 6 proportional reasoning. Topic B continues as students use given rules (e.g., multiply by 2 and then add 3) to generate ordered pairs, plot points, and investigate relationships in both graphs and tables, Patterns in the resultant coordinate pairs are analyzed, leading students to discover that such rules produce collinear sets of points. Students next generate two number patterns from two given rules, plot the points, and analyze the relationships within the sequences of the ordered pairs (**5.4C**). Patterns continue to be the focus as students analyze the effect on the steepness of the line when the second coordinate is produced through an addition rule as opposed to a multiplication rule (**5.4D**). Students also create rules to generate number patterns, plot the points, connect those points with lines, and look for intersections. To round out the topic, students use coordinate planes to solve real-world problems.

In previous grades and modules, students represented data with dot plots, including data sets of measurements in fractions, and stem-and-leaf plots. In Topic C, students collect data with measurements in decimals and represent this data on dot plots or stem-and-leaf plots (**5.9A**). Students learned to graph points in the coordinate plane in Topic A. They extend this work as they collect, organize, display, and interpret data using scatterplots (**5.9B**). Students then apply their knowledge of scatterplots to solve one-and two-step problems (**5.9C**).

Topic D provides an opportunity for students to encounter complex, multi-step problems requiring the application of concepts and skills mastered throughout the Grade 5 curriculum. They use all four operations with both whole numbers and fractions in varied contexts. The problems in Topic D are designed to be non-routine, requiring students to persevere to solve them. While wrestling with complexity is an important part of Topic D, the true strength of this topic is derived from the time allocated for students to construct arguments and critique the reasoning of their classmates. After students have been given adequate time to ponder and solve the problems, two lessons are devoted to sharing approaches and solutions. Students partner to justify their conclusions, communicate them to others, and respond to the arguments of their peers.

In the final topic of Module 6 and, in fact, *A Story of Units®*, students spend time applying their knowledge of the coordinate plane in non-routine ways and producing a compendium of their learning. They not only reach back to recall learning from the very beginning of Grade 5, but they also expand their thinking by exploring such concepts as the Fibonacci sequence. Students solidify the year's learning by creating and playing games, exploring patterns as they reflect on their elementary years. All materials for the games and activities are then housed for summer use in boxes created in the final two lessons of the year.

## Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Lessons 5 and 6 share the same objective and can be consolidated. Lesson 11 is not part of the Grade 5 TEKS and therefore may be omitted.

Topics D and E occur after the End of Module Assessment and are optional. However, they afford students the opportunity to reflect on all the learning they have experienced in Grade 5 and throughout *A Story of Units*. These Topics serve as both an excellent culmination to elementary school and a meaningful bridge to middle school.

# Focus Grade Level Standards

## Number and Operations

**The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:**

**5.3D** represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;

**5.3E** solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;

**5.3F** represent quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using objects and pictorial models, including area models;

**5.3G** solve for quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm;

**5.3I** represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models;

**5.3K** add and subtract positive rational numbers fluently;

**5.3L** divide whole numbers by unit fractions and unit fractions by whole numbers.

## Algebraic Reasoning

**The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:**

**5.4C** generate a numerical pattern when given a rule in the form $y = ax$ or $y = x + a$ and graph;

**5.4D** recognize the difference between additive and multiplicative numerical patterns given in a table or graph;

**5.4H** represent and solve problems related to perimeter and/or area and related to volume.

## Geometry and Measurement

**The student applies mathematical process standards to identify locations on a coordinate plane. The student is expected to:**

**5.8A** describe the key attributes of the coordinate plane, including perpendicular number lines (axes) where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the *x*-coordinate, the first number in an ordered pair, indicates movement parallel to the *x*-axis starting at the origin; and the *y*-coordinate, the second number, indicates movement parallel to the *y*-axis starting at the origin;

**5.8B** describe the process for graphing ordered pairs of numbers in the first quadrant of the coordinate plane;

**5.8C** graph in the first quadrant of the coordinate plane ordered pairs of numbers arising from mathematical and real-world problems, including those generated by number patterns or found in an input-output table.

### Data Analysis

**The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:**

**5.9A** represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-and-leaf plots;

**5.9B** represent discrete paired data on a scatterplot;

**5.9C** solve one- and two-step problems using data from a frequency table, dot plot, bar graph, stem-and-leaf plot, or scatterplot.

## Foundational Standards

**The student is expected to:**

**4.4H** solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders;

**4.5B** represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule representing the relationship of the values in the resulting sequence and their position in the sequence;

**5.3H** represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations;

**5.3J** represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1/3 \div 7$ and $7 \div 1/3$ using objects and pictorial models, including area models;

**5.4E** describe the meaning of parentheses and brackets in a numeric expression.

# Focus Mathematical Process Standards

**The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**

**MPS(A)** apply mathematics to problems arising in everyday life, society, and the workplace;

**MPS(B)** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

**MPS(E)** create and use representations to organize, record, and communicate mathematical ideas;

**MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas;

**MPS(G)** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

# Overview of Module Topics and Lesson Objectives

| TEKS | ELPS | | Topics and Objectives | Days |
|---|---|---|---|---|
| **5.8A**<br>**5.8B** | 1.C<br>2.A<br>2.E<br>3.A<br>3.E<br>3.H<br>4.I<br>5.G | A | **Coordinate Systems**<br>Lesson 1: Construct a coordinate system on a line.<br>Lesson 2: Construct a coordinate system on a plane.<br>Lessons 3–4: Name points using coordinate pairs, and use the coordinate pairs to plot points.<br>Lessons 5–6: Investigate patterns in vertical and horizontal lines, and interpret points on the plane as distances from the axes. | 6 |
| **5.4C**<br>**5.4D**<br>**5.8C**<br>5.4E<br>5.8A<br>5.8B | 1.D<br>1.E<br>1.H<br>2.E<br>2.I<br>3.E<br>3.G<br>4.I<br>5.B<br>5.G | B | **Patterns in the Coordinate Plane and Graphing Number Patterns from Rules**<br>Lesson 7: Plot points, use them to draw lines in the plane, and describe patterns within the coordinate pairs.<br>Lesson 8: Generate a number pattern from a given rule, and plot the points.<br>Lesson 9: Generate two number patterns from given rules, plot the points, and analyze the patterns.<br>Lesson 10: Compare the lines and patterns generated by addition rules and multiplication rules.<br>Lesson 11: Analyze number patterns created from mixed operations.<br>Lesson 12: Create a rule to generate a number pattern, and plot the points.<br>Lesson 13: Use coordinate systems to solve real-world problems. | 7 |
| | | | Mid-Module Assessment: Topics A–B (assessment 2 days, return 1 day, remediation or further applications 1 day) | 4 |

| TEKS | ELPS | Topics and Objectives | | Days |
|---|---|---|---|---|
| **5.9A**<br>**5.9B**<br>**5.9C** | 1.A<br>1.E<br>2.C<br>2.E<br>3.E<br>3.G<br>3.H<br>3.I<br>4.I<br>5.G | C | **Data Representation and Analysis**<br>Lesson 14: Collect and represent data using dot plots.<br>Lesson 15: Represent data using stem-and-leaf plots.<br>Lesson 16: Collect and represent discrete paired data on a scatterplot.<br>Lesson 17: Describe patterns and solve problems by using scatterplots.<br>Lesson 18: Solve problems using data. | 5 |
| | | | End-of-Module Assessment: Topics A-C (assessment 2 days, return 1 day, remediation or further applications 1 day) | 4 |
| **5.3D**<br>**5.3E**<br>**5.3F**<br>**5.3G**<br>**5.3I**<br>**5.3K**<br>**5.3L**<br>**5.4H**<br>5.3H<br>5.3J | 1.G<br>1.H<br>2.E<br>2.F<br>3.C<br>3.E<br>3.F<br>3.G<br>3.J<br>4.I<br>5.G | D | **Multi-Step Word Problems**<br>Lesson 19: Make sense of complex, multi-step problems, and persevere in solving them. Share and critique peer solutions.<br>Lesson 20: Make sense of complex, multi-step problems, and persevere in solving them. Share and critique peer solutions.<br>Lesson 21: Make sense of complex, multi-step problems, and persevere in solving them. Share and critique peer solutions.<br>Lesson 22: Make sense of complex, multi-step problems, and persevere in solving them. Share and critique peer solutions.<br>Lesson 23: Make sense of complex, multi-step problems, and persevere in solving them. Share and critique peer solutions. | 5 |

| TEKS | ELPS | Topics and Objectives | | | Days |
|---|---|---|---|---|---|
| | 1.A<br>1.E<br>1.H<br>3.C<br>3.E<br>3.G<br>3.J<br>4.I<br>5.G | E | **The Years in Review: A Reflection on *A Story of Units*** | | 11 |
| | | | Lesson 24: | Draw symmetric figures on the coordinate plane. | |
| | | | Lesson 25: | Plot data on line graphs and analyze trends. | |
| | | | Lesson 26: | Solidify writing and interpreting numerical expressions. | |
| | | | Lesson 27: | Solidify writing and interpreting numerical expressions. | |
| | | | Lesson 28: | Solidify fluency with Grade 5 skills. | |
| | | | Lesson 29: | Solidify the vocabulary of geometry. | |
| | | | Lesson 30: | Solidify the vocabulary of geometry. | |
| | | | Lesson 31: | Explore the Fibonacci sequence. | |
| | | | Lesson 32: | Explore patterns in saving money. | |
| | | | Lesson 33: | Design and construct boxes to house materials for summer use. | |
| | | | Lesson 34: | Design and construct boxes to house materials for summer use. | |
| **Total Number of Instructional Days** | | | | | **42** |